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RAYTHEON

ELECTRONIC PROGRESS

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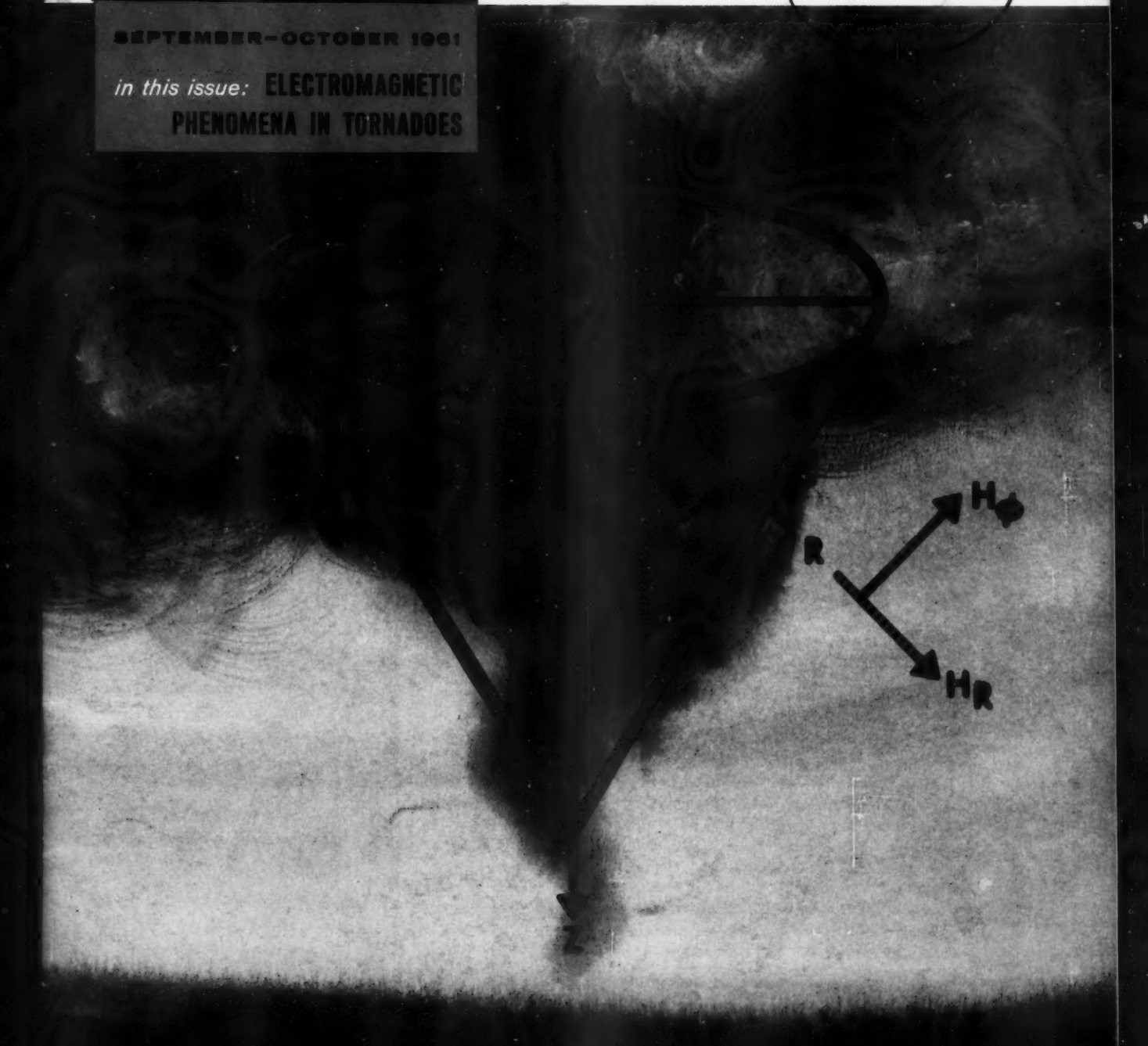
SEPTEMBER–OCTOBER 1961

in this issue: ELECTROMAGNETIC PHENOMENA IN TORNADOES



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RAYTHEON

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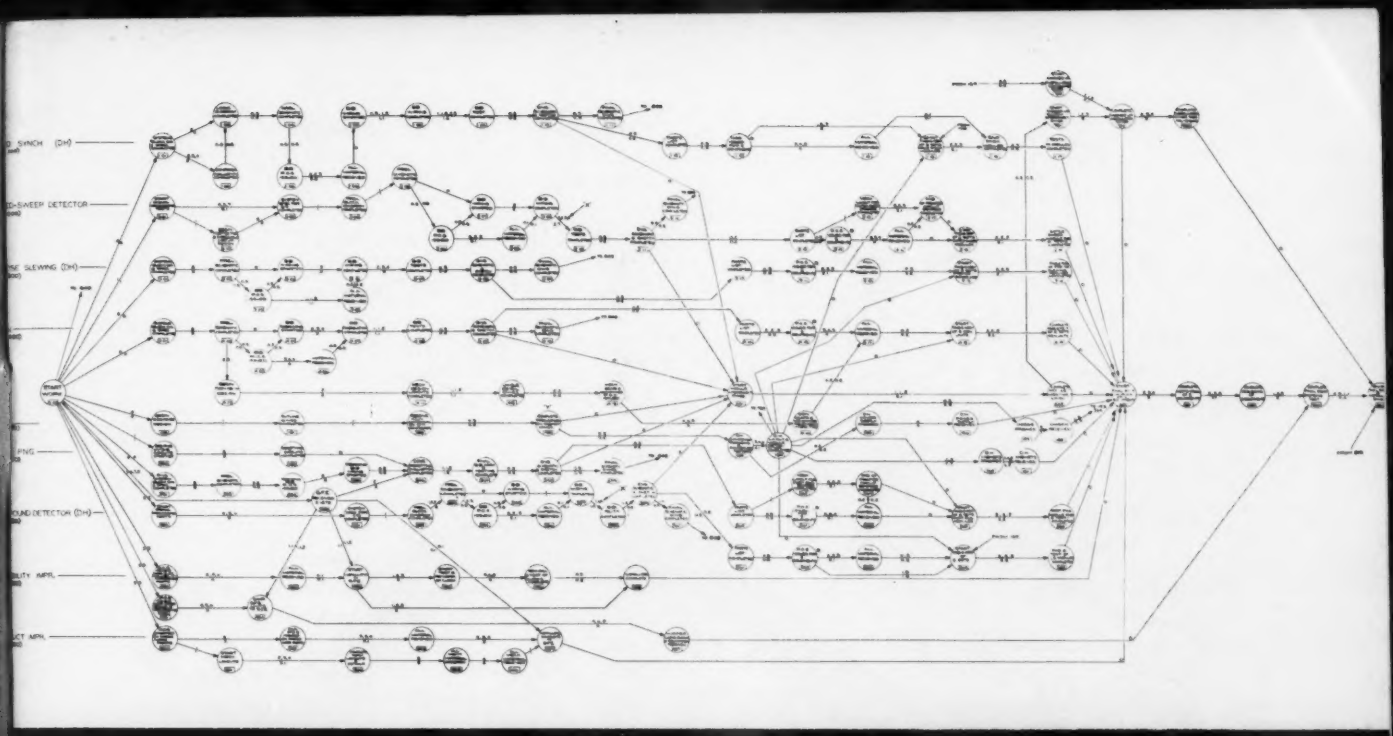
COVER

The growing interest of scientists in the causal behavior of severe weather phenomena is dramatized by superimposing a magnetic field diagram on the classic funnel cloud of a tornado. The electrical characteristics of tornadoes are explored in this issue.



PERT

PROGRAM EVALUATION REVIEW TECHNIQUE

by Robert W. Miller

THE PROBLEM of planning and executing programs with a high development engineering content has increased by at least an order of magnitude during the last decade. The techniques of Frederic W. Taylor and Henry L. Gantt, introduced during the early part of the century, and used so successfully for large-scale production operations, seem inappropriate in the current age of "massive engineering." Indeed the validity of conventional planning, budgeting, and control approaches, which are ultimately based upon input data of the Taylor-Gantt variety, has been increasingly questioned by industrial organizations engaged in development effort, both inside and outside the defense industry.

The brunt of the problem, however, clearly centers upon the United States government military and industrial complex, as it is concerned with the planning and execution of national defense and space programs. In the defense area, some quantitative indication of the seriousness of the problem can be obtained from a study by A. W. Marshall and W. H. Meckling of the Rand Corporation in 1959 entitled "Predictability of the Cost, Time and Success of Development." In this study, the cost and schedule histories of 22 major items of military equipment are analyzed. After some difficult but conservative adjustments to their basic data, the authors point out that the ratio of the *latest cumulative average*

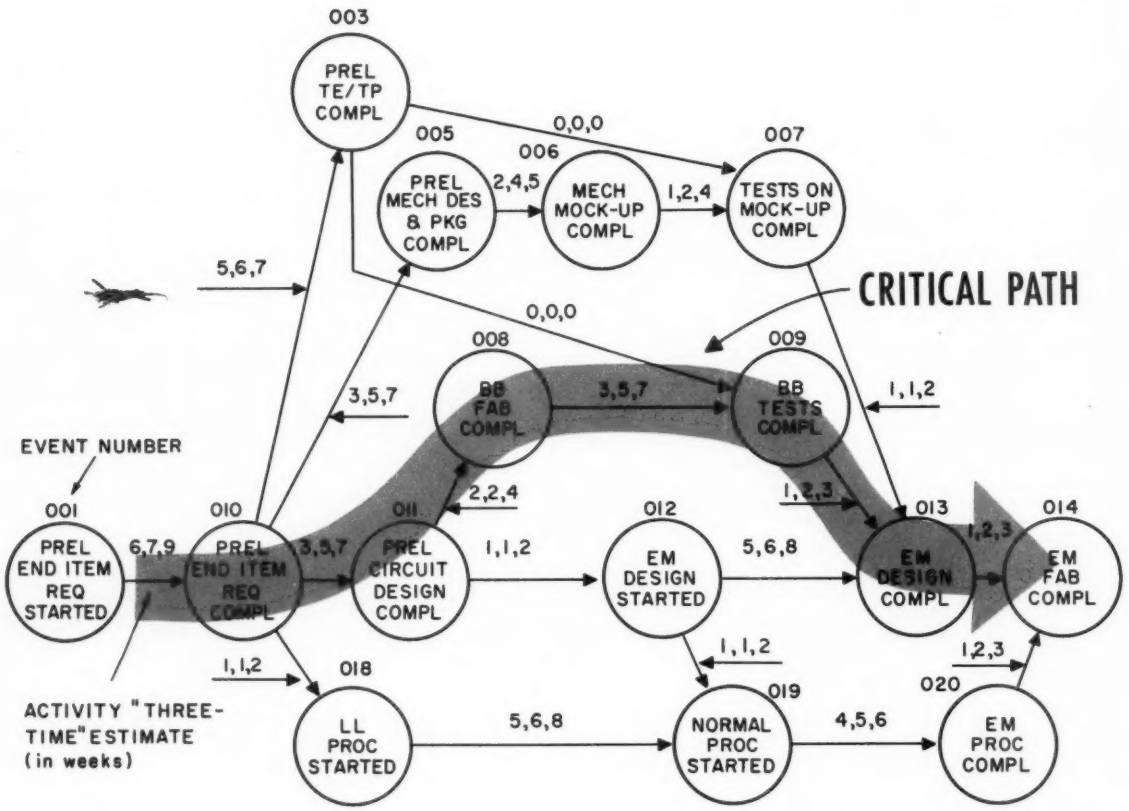


Figure 1. Portion of a typical operating network

cost of production to the *earliest available estimate for a given program* ranged from 1.1 to 7.7, with the average being a factor of 2.4. Although Marshall and Meckling give production figures only in this study, they indicate a similar range of variance for development costs. In the area of time performance, slippage in availability of first operational equipment ranged from ⅓ of a year to five years, with the average being two years. As might be expected, the data vary according to the amount of technological advance involved in the program, with the new missile and space programs having the highest figures.

It should be pointed out that Marshall and Meckling's data encompass the total complex of circumstances surrounding a military development program, including changes in performance specifications and funding levels. Their study indicates, without exception, the existence of very large optimistic biases of the cost and time required to achieve a given equipment performance level. In view of these figures the situation should clearly be regarded as serious by all parties involved, particularly in view of the fact that an ever-increasing amount of the nation's Gross National Product is devoted to the state-of-the-art programs of the space age.

It is in this climate that we have seen the recent emergence of some new concepts of planning and control for development-oriented programs, an outstanding example of which is the Navy's PERT—first implemented on the Polaris program in 1959.

Because of its participation in the Polaris program, Raytheon was exposed to the PERT system at an early date, and the first complete PERT network was established on the Independent SESCO program in the Equipment Division in March 1960. Since that time, PERT has been implemented on a number of programs within the Government Equipment & Systems Group, including the Polaris Guidance Package, Mauler, APS-104, ARPAT, ECM Penetration Aids, B-52 Decoy, Advent, PCM, and Bright Display. It is a significant fact that with some of the latter programs PERT is

not a contractual requirement, and though the complete methodology has not always been used, the significant features of network development and critical path and slack analysis have been preserved.

In addition, PERT is an essential feature of many Raytheon proposals, particularly those which involve effort on major systems programs. Its specification as a contract requirement has spread to the Air Force, Army, and NASA. Although currently the degree of emphasis placed on PERT varies among agencies, one can predict its use on all development-oriented government programs within the next few years.

Theory

PERT, at present, is restricted to the time domain, and as promulgated by the Navy, has the following basic requirements:

1. All of the individual tasks to complete a given development program must be visualized in a clear enough manner to be put down in *network* form. Emphasis is placed on defining *events* and *activities* of the network with sufficient precision so that there is no difficulty in monitoring actual accomplishments as the program proceeds. Figure 1 shows a portion of a typical operating network.
2. *Events* and *activities* must be sequenced on the network under a highly logical set of ground rules which allows the determination of important *critical and subcritical paths.* These ground rules include the fact that no successor event can be considered completed until all of its predecessor events have been completed, and no looping is allowed, i.e., no successor event can have an activity dependency which leads back to a predecessor event.
3. Time estimates are made for each activity of the network on a three-way basis, i.e., *earliest, most likely, and pessimistic* elapsed time figures are estimated by the person or persons most familiar with the effort involved. The three time estimates

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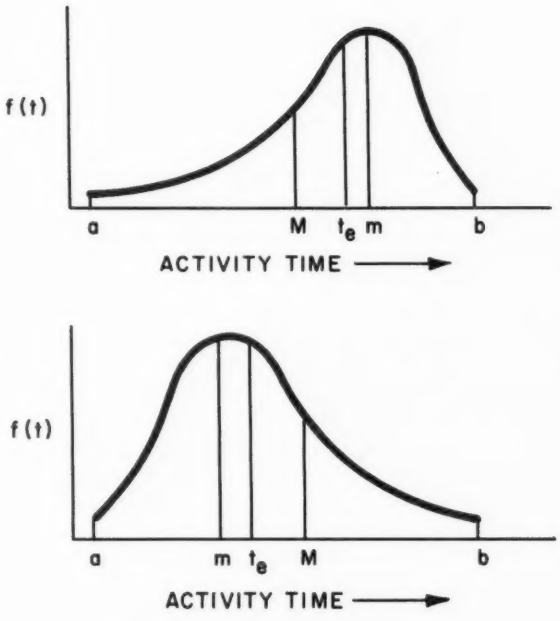


a = OPTIMISTIC TIME

m = MOST LIKELY TIME

b = PESSIMISTIC TIME

$M = \text{MID-RANGE } \left(\frac{a+b}{2}\right)$

$t_e = \text{EXPECTED TIME } \left(\frac{a+4m+b}{6}\right)$

$T_e = \sum t_{e_1} + t_{e_2} + \cdots t_{e_n}$

$\sigma^2 = \text{VARIANCE OF } t_e = \left(\frac{b-a}{6}\right)^2$

Figure 2. Beta distribution of activity times

are required as a gauge of the *measure of uncertainty* of the activity, and represent full recognition of the probabilistic nature of many of the tasks in development-oriented programs. It is important to note, however, for the purposes of computation and reporting, the three-time estimates are reduced to single *expected time* t_e and a *variance* σ^2.

4. Depending on the size and complexity of the network, computer routines are available to calculate the critical path through the network, and also the amount of *slack* (i.e., extra time available) for all events and activities not on the critical path. A *negative slack* condition can prevail when a calculated end date does not achieve a program date objective which has been established on a prior, and often arbitrary, basis.

Interpretation of the concepts of *earliest, most likely,*

and *pessimistic* elapsed times has varied over the past two years. Definitions which the author feels represent a useful consensus are:

Earliest: An estimate of the *minimum* time an activity will take, a result which can be obtained only if unusual good luck is experienced, i.e., "everything goes right the first time."

Most Likely: The *most realistic* estimate of the time an activity will take, representing a "normal" combination of successes and failures which would occur most often if the activity could be repeated a number of times under similar circumstances.

Pessimistic: An estimate of the *maximum* time an activity will take, a result which can occur only if unusually bad luck is experienced. It should reflect the possibility of initial failure and fresh start, but should not be influenced by such factors as "catastrophic events," such as strikes, fires, power failures, etc., unless these hazards are inherent to the activity.

In analyzing the three time estimates, it is clear that the earliest and pessimistic time should occur least often, and the most likely should occur most often. Thus it is assumed that the most likely time represents the peak or modal value of a frequency distribution, but is, however, free to move between the two extremes. These characteristics are best described by the *Beta* distribution, which is shown in two different conditions in Figure 2. The probability density of this distribution is described by:

$$f(t) = K(t-a)^{\alpha}(b-t)^{\lambda} \quad , \qquad (1)$$

where

$$K = \frac{1}{(b-a)^{(\alpha+\lambda+1)}\,\beta(\alpha+1,\lambda+1)} \qquad (2)$$

and α and λ are integral multiples of ½. From this distribution, the final approximations to the expected time t_e and variance σ^2 were written as:[1]

$$t_e = \frac{1}{3}(2m + M) \qquad (3)$$

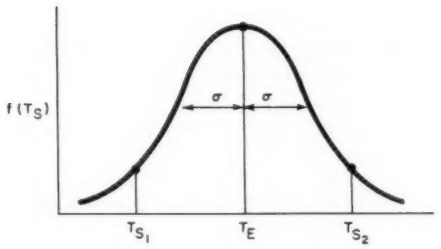
$$= \frac{1}{3}\left(2m + \frac{a+b}{2}\right)$$

$$= \frac{a+4m+b}{6} \qquad (4)$$

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2 \quad . \qquad (5)$$

Equation 3 indicates that t_e should be interpreted as the weighted mean of m (most likely) and M (mid-range) estimates, with weights of 2 and 1, respectively. In other words, t_e is located one-third of the way from the modal to the mid-range values. The weights of 2 and 1 in the approximate formula are subject to question, but have been widely used in actual practice, and

[1]PERT Summary Report, Phase 1, Special Projects Office, Department of the Navy, Washington, D.C. (July 1958).



$$T_E = \Sigma t_{e_1} + t_{e_2} + t_{e_n}$$

$$\sigma^2(T_E) = \Sigma\,\sigma^2(t_e) + \sigma^2(t_{e_2}) + \sigma^2(t_{e_n})$$

T_{S_1} = SCHEDULE TIME (earlier than T_E)

T_{S_2} = SCHEDULE TIME (later than T_E)

$$\frac{T_{S_1} - T_E}{\sigma} = -1.5\sigma\,,\ P_r\ \text{(accomplishment)} = 0.13$$

$$\frac{T_{S_2} - T_E}{\sigma} = +1.5\sigma\,,\ P_r\ \text{(accomplishment)} = 0.87$$

Figure 3. Probability of accomplishing schedule, T_S

at this point seem appropriate enough in view of the inherent accuracy of the input data.

A special feature of PERT is the possibility of calculating the probability of meeting a given, or arbitrarily established schedule date T_S (see Figure 3). While one may argue about the importance or efficacy of this aspect of PERT, the theory is summarized as follows:

Based on the Central Limit Theorem one can conclude that the probability distribution of times for accomplishing an activity may be approximated by the normal distribution, and that this approximation approaches exactness as the number of activities becomes large (for example, more than 10 activities along a given path). Thus we may define a curve as shown in Figure 3 where:

$$f(T_S) = \frac{1}{\sigma\sqrt{2\pi}}\,\frac{e^{-1/2\,(T_S - T_E)^2}}{\sigma} \quad .$$

The probability of meeting the T_S date when given T_E and σ for a chain of activities (as shown in Figure 3), is defined as the ratio of the area under the curve to the left of T_S over the area of the entire curve. The final result is that the difference between T_S and T_E, expressed in units of σ, i.e.,

$$\frac{T_S - T_E}{\sigma}$$

will yield a value for the probability of accomplishing T_S by use of the normal probability distribution table.

In practice, the more important feature of PERT methodology is the calculation of critical path and slack times. These calculations for the network example of Figure 1 are shown in Figure 4 (Slack Order Report), which is perhaps one of the most important output reports of the PERT system. The calculations shown in

Figure 4 may be verified as follows:

T_E, Expected Event Time, is found by summing all t_e's and choosing the highest figure, or longest path leading into any one event; T_L, Latest Event Time, is found by a reverse process which involves determining the latest allowable times for all events so as not to affect critical path event times;

$T_L - T_E$, Slack, represents the amount of time an event can slip and still not affect the critical path. As seen in Figure 4, slack is equal to zero along the critical path, and all other events have positive slack. A *negative* slack condition can prevail when there is a Scheduled Event Time T_S which is earlier than T_E for any event.

Other output reports, i.e., Event Order and Calendar Time Order, are available in the PERT system. The actual utilization of the system involves review and action by appropriate managers, generally on a bi-weekly basis. Because time prediction and performance data is available from PERT in a *highly-ordered* fashion, managers are given the opportunity to concentrate on the important critical and subcritical path activities, either by applying new resources or additional funds, particularly from activities which can afford it because of their slack condition. Alternatively, the manager is faced with the problem of re-evaluating the sequencing of activities along a critical path, and placing those activities which were formerly connected in a series, on a parallel or overlapping basis with the concomitant trade-off risks involved.

The PERT system requires constant updating and re-analysis, i.e., it takes recognition of the fact that outlook for individual activities in a development-oriented program are in a constant state of flux, and that management must be constantly concerned with problems of re-evaluation and re-programming. A highly systematized method of input reporting has been developed to handle this aspect of PERT. (see Figure 5)

PERT SYSTEM Airborne Computer—Slack Order
Date 7/12/61 **Week 0.0**

EVENT	T_E	T_L	$T_L - T_E$ (slack)	T_S	P_r	Scheduled Date
001	0.0	0.0	0			
010	7.2	7.2	0			
011	12.2	12.2	0			
008	14.5	14.5	0			
009	19.5	19.5	0			
013	21.5	21.5	0			
014	23.5	23.5	0			
020	20.6	21.5	+0.9			
019	15.6	16.5	+0.9			
012	14.4	15.3	+0.9			
018	9.4	10.3	+0.9			
007	18.2	20.3	+2.1			
006	16.0	18.1	+2.1			
005	13.2	14.3	+2.1			
003	14.2	14.5	+5.3			

T_E = Expected Date
T_L = Latest Date
T_S = Scheduled Date
P_r = Probability of Meeting T_S

Figure 4. Slack order report

Advantages of PERT

Perhaps the major advantage of the PERT technique is that the planning effort required to create a valid network represents a major contribution to the definition and ultimate successful control of a development program. It may surprise some that network creation and critical path analysis do in fact reveal interdependencies and problem areas which are either not obvious or well defined by conventional planning methods. The development of the network is a fairly demanding task, and is a sure-fire indicator of an organization's ability to visualize the number, kind, and sequence of activities needed to execute development programs.

Another advantage of PERT is the matter of three-way rather than single-time estimating, in the case where there is a significant amount of uncertainty. While introducing a complicating feature, the three-way estimate does give recognition to those realities of life which are commonplace in development programs. It is assumed that the three-way estimate will result in a greater degree of honesty and accuracy in time forecasting, and as a minimum, will allow the decision-maker a better opportunity to evaluate the degree of uncertainty involved in a schedule. If he is statistically sophisticated, the decision-maker may even wish to examine the standard deviation and probability of accomplishment data which is available as a feature of PERT. At the other end of the spectrum, if there is a minimum of uncertainty in the minds of personnel estimating individual activity times, the single-time approach may be used, while retaining all the advantages of network analysis.

Finally, a third advantage of PERT is its common language feature which allows a large amount of data to be presented in a highly ordered fashion. It can be said that PERT represents advent of management-by-exception principle in an area of planning and control where it has not existed with any real degree of validity.

Problems of Pert

In PERT methodology an available set of resources including manpower, facilities, and equipment is either known or must be assumed at the time of making the three estimates. For example, it is good practice to make special notations directly on the network when some special condition, e.g., a 48-hour rather than a 40-hour week is assumed. Experience has shown that, when a well thought-through network is developed in sufficient detail, the first activity time estimates made are as accurate as any, and that these should not be changed unless a new application of resources or trade-

PERT REPORT OF TIME INTERVAL ESTIMATES & PROGRESS				CLASSIFICATION:					Revision No. 2 13 February 1959	
From: (Name & Location of Contractor)				To:					Flow Chart No. / Contract No.	Report Period From: / To:
For office use only (A)				Activity Identification		Time Interval Estimates			Completion Date	Remarks
				Beginning Event No.	Ending Event Nc	Optimistic (weeks) *	Most Likely (weeks) *	Pessimistic (weeks) *		
(1)	(2)	(3)	(4)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
12	13—16		17	18—26	34—42	44—47	48—51	52—55	60—65	
									Mo. Day Yr.	
1				010	003	5.0	6.0	7.0	— — —	New Activity
1				003	007	0	0	0	— — —	New Activity
1										
2				010	018	1.0	1.0	2.0	— — —	Re-estimated Activity (Change)
2				018	019	5.0	6.0	8.0	— — —	Re-estimated Activity (Change)
5										
Signature of Responsible Official:				Date Signed:					CLASSIFICATION:	

*Columns D, E, and F. These estimates should be given for the full activity even though the activity has already started.

Figure 5. Input transaction documents

off in performance characteristics is specifically determined. A further caution is that the first time estimates should not be biased by some arbitrarily-established schedule objective, or the assumption that a particular activity does not appear to be on a critical path. Schedule-biasing of this kind, while it obviously cannot be prevented, clearly atrophies some of the main benefits of the technique, and is more quickly discovered under PERT than any other method. In the experience with PERT to date, it is this factor more than any other which determines a bad versus a good application, assuming that the organization has basic capability in the technology of the development program involved.

Because of the necessity for manpower and resource assumptions PERT works best in project-structured organizations, where the level of resources and available facilities are more clearly known to the estimator. PERT does not itself explicitly resolve the problem of multi-project planning and control through common resource centers (i.e., shops or drafting rooms). There is general recognition of this problem, and considerable effort is being devoted to a more complete approach to the multi-project scheduling problem. Meanwhile, it is generally necessary to use subsidiary loading analysis in the case of common resource centers, making priority assumptions, and using the resulting data on either a three-time or single-time basis for those portions of the network which are affected. It should be pointed out, however, that in terms of actual experience with PERT, the process of network development forces more problems of resource or loading analysis into the open for resolution than do conventional planning methods.

Although PERT has been characterized as a new management control approach for R&D effort, it has perhaps been most usefully applied in development rather than research situations; i.e., programs where there is typically a great deal of interconnection between the activities of a network, or interface connections between different networks. Certainly network development and critical path analysis are not too appropriate for the pure research situation, where the capabilities of small numbers of individuals with highly specialized talents are being utilized at a "constant rate," and

their activities have no significant dependence upon other elements of the organization.

One of the most frequently raised objections to PERT is the cost of its implementation. A fundamental point to examine when this issue is raised is whether or not a currently established scheduling system is giving value commensurate with its cost, or indeed if it is even used at all effectively to pinpoint and control problem areas. It is quite true that, by the very nature of its logical requirements for networking, the PERT approach calls for a higher degree of planning skill and a greater amount of detail than is the case with most scheduling systems. The cost of implementing PERT has been estimated at anywhere from 0.1% to 1.0% of total engineering product cost.

It is perhaps more appropriate to view the implementation of PERT as costing *initially* something on the order of twice a conventional scheduling system, and this figure will vary significantly with such factors as the degree of planning capability already available, the present effectiveness and homogeneity of the organization, and the amount and quality of the PERT indoctrination given. The advocates of PERT are quick to point out that the savings achieved through better utilization of resources during the development phase, and the very considerable savings that result during the manufacturing build-up phase through better time performance on the part of development engineering, are more than worth its initial implementation costs. Such conclusions are undoubtedly valid in those cases where management carefully monitors critical path movements, and undertakes all the action steps within its power to alleviate problem areas at the earliest possible stage.

Because of the considerable impact of PERT in many organizations where detailed planning has not had major emphasis, a trend has recently developed which can be characterized as the problem of *model networking*. It is quite true that in each industry involved in the age of massive engineering, there have grown up typical or established patterns of carrying out development work. In fact, some major defense organizations utilizing the PERT system have published dictionaries containing *standard event* definitions. While these refinements have a great deal of merit, and in particular alleviate problems of inconsistency that inevitably arise in the handling of the large amounts of data involved in PERT, there is a danger that they can obviate one of the real objectives of the technique—namely, obtaining a valid network which is meaningful to the person or persons who will actually execute the work. The introduction of model networks should be handled with caution, and always allow for the possibility of modification which will match the realities of the development program.

So too, the introduction of *master plan networks* and the *top-down structuring* of networks for large programs, while a very necessary development from the point of view of the ultimate managers of such programs, should be handled with some degree of caution and at least a philosophy of flexibility. The cardinal principle is that a management control structure is no better than the adequacy and accuracy of the data at its base. It appears likely that in the future we shall experience a trend where the top-down structuring approach which is already evident on some major military programs will increase, but be subject to reconfirmation or even realignment at the level of industry, contingent upon the development of actual operating networks.

A final problem, and one that is often claimed as a disadvantage of the PERT technique, is its lack of applicability to manufacturing. As has been stated, PERT deals in the time domain only, and does not contain the quantity information required in most production control operations. Nevertheless, PERT can be, and has been, used very effectively through the preliminary manufacturing phases of production prototype or pilot model construction, and assembly and test of final production equipments which are still high on the learning curve. After these steps, established production control techniques which bring in the quantity factor are generally more applicable. Many programs, however, which have a high development content never leave the preliminary manufacturing stage, and therefore there is considerable effort going forward at this time to integrate the techniques of PERT within some of the established methods of production control, such as Line-Of-Balance or similar techniques.

Computer or No Computer

As a result of the Navy's successful application of PERT on the Polaris Program, and other similar applications, there is a common impression that the technique is only applicable when large-scale data processing equipment is available. This is certainly true for large networks or aggregations of networks, and is simply another way of saying it is desirable to have a computer to handle the PERT technique when a large volume of data is available, as is the case for most other valid computer applications. At Raytheon the Bedford IBM 704 and LPG-30 computers have been programmed for PERT. The Equipment Division is using a special program developed for Mauler, using rented IBM 7090 computer time, which contains much more information than is found in standard PERT outputs. Another program known as LESS (Least Cost Estimating and Scheduling System) is available on the Corporate IBM 650 Computer, but this yields outputs which were basically developed for the construction industry, and not generally familiar to personnel indoctrinated in PERT.

Probably equally significant is the fact that manual methods have been developed by those organizations which have become convinced of the usefulness of the method. These manual methods range from simple in-

spection on small networks, to more organized but clerically-oriented routines for determination of critical path, subcritical path, and slack times on networks ranging from 50 to several hundred events. It is therefore a truism that the PERT technique can be applied equally successfully to small programs whenever the degree of interconnection and problems of uncertainty warrant it. For those orgainzations practiced in the technique, the creation of small networks, the time estimates, and their reduction to critical path and slack analyses can be in the order of hours, and therefore computerization is not necessary for these applications. The PERT illustration on page one portrays a 150-event network which was developed at Submarine Signal Operations in less than a day. An additional day was necessary for corroboration of time estimates, and manual computation required only two hours. Final drafting for presentation to the customer required an additional two days, and this particular network has remained basically unchanged since that time.

Pert Extensions

Variations of the PERT system to accommodate the multi-project and manufacturing situations have already been mentioned, and these are merely representative of a basic movement to *extend* the PERT approach into the areas of manpower, cost, and the equipment performance variable. The ultimate objective of these efforts is to quantify the trade-off relationships which constantly come up in development programs, but which are rarely acted upon with explicit data in hand.

Though none of these extensions have as yet attained the relative degree of maturity and acceptance of PERT, anyone familiar with the current scene will be impressed by the amount of activity taking place throughout the country in the area within both the military and industry. At least 30 different code names or acronyms representing variations of the new management controls have come to the attention of the author. One healthy offset to this particular trend is the fact that the U.S. Air Force has officially withdrawn its code name PEP (Program Evaluation Procedure), which was an equivalent for PERT. With regard to the new extensions, however, one can predict a greater range of differences compared to PERT, unless developments sponsored by DOD become accepted as the standard, as was the case with *PERT-time*.

Pert Cost

Much of the current research effort on the new management controls is concentrated on the problem of cost. This is probably an expression of several facts well-understood by most managers of development programs, namely that:

1. The job costing structures generally found in industry need a great deal of interpretation to relate *actual costs* to *actual progress*, and are rarely, if ever, directly related to the details of the scheduling plan.
2. Cost constraints, either in the form of manpower shortages or funding restrictions, have a great deal to do with the success with which the development program can be managed.

It seems clear that both these problems must be solved in any valid PERT cost approach.

In the first of these problems, an explicit relationship must be established between the job cost structure and the time network, either on a one-to-one basis for each network activity, or for a designated grouping of such activities, i.e., sub-networks. As a minimum, it seems clear that more detailed job cost structures are required than are currently in general use, although this requirement should present no serious limitation for organizations possessing modern data processing methods and equipment.

As regards the development of actual cost figures from the network, an estimate of manpower requirements segregated by classification is usually regarded as the

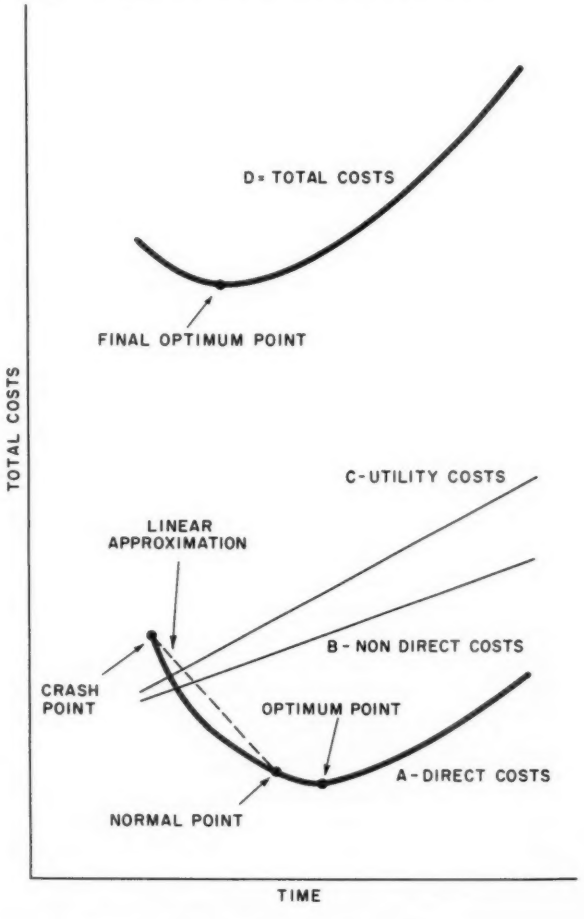


Figure 6. Assumed time-cost relationships for a job

easiest place to start, since these requirements were assumed to be known at the time the network was established. In fact, however, the actual summation of such data often reveals a manpower or funding restriction problem, and forces a re-planning cycle, if no alternatives are available. At the same time, it may reveal inefficiencies in the loading outlook for personnel, which can be removed by proper utilization of slack activities.

Another problem is the handling of non-labor costs, such as material costs, which are often aggregated in a manner quite different than would result from analysis of a time network. An example is the tendency to buy materials on common Purchase Orders for a number of different engineering prototypes, each one of which represents a distinct phase of progress in the program.

Another problem is that of coordination and control efforts, such as those carried out by project or systems engineering, which are often not indicated on time networks unless they result in specific outputs. All of these problems can be handled and, as a result, a number of different approaches are being attempted in the development of *cost networks*, which have as their objective the association of at least a single cost estimate with a known activity or chain of activities.

The ultimate objective of the identification of time and cost networks is not only for improvement in control, but the opportunity to assess time-cost trade-off situations. It is generally assumed that the fundamental relationships between time and cost are as portrayed in Figure 6. Curve A represents *total direct costs* versus time, and the "U" shape of the curve results when there is an "optimum" time-cost point for any activity or job, and when total costs increase with any effort to accelerate or delay the job away from this point. The practical difficulties of quantifying such curves, particularly on programs with a high development content, are very real indeed. However, it should be noted that certain segments of the construction industry are already using such a time-cost relationship, even though in a rather specialized manner. In the LESS (Phase III) program, an assumption is made that there is a *normal* job time (which might or might not coincide with the theoretical optimum), and that from this normal time, costs increase linearly to a *crash* time (indicated in Figure 6), which represents the maximum acceleration the job can stand. Based upon these assumptions, a complete mathematical approach and computer program have been developed which result in *maximizing job acceleration for minimum cost.* The process involves shortening the critical path or paths by operating upon those activities which have minimum time-cost slopes.

The assumption that time-cost data is available for each activity, or that it takes on a simplified form, is one of the fundamental problems facing the use of such an approach on development programs. In particular, at the planning stage of such programs, it is usually very difficult to determine time-cost relationships in any explicit manner, either for individual activities or aggregates of activities. At the planning stage, there are often good arguments for characterizing the relationships as non-linear, flat, decreasing, or more likely, as a range of cost possibilities. It is known that solutions for the technical handling of these problems have recently been developed, however.

Curve B of Figure 6 indicates *total non-direct costs*, which are assumed to increase linearly with time. It is clear that accounting practices will have to be reviewed to provide careful and probably new segregations of direct versus non-direct costs, for application in this frame of reference. It also seems clear that the problem of properly segregating such non-direct costs will be greater in the multi-program situation.

Curve C is a representation of a *utility cost curve*, which is needed to complete the picture for total time-cost optimization, indicated as the *final optimum point* on Curve D. The utility cost curve attempts to quantify the penalty for *not accomplishing the job at the earliest possible time*, and is also shown as a linear function increasing with time. The difficulties of determining such a curve for a military development program, either in terms of its shape or dollar value, should be obvious. It is significant to note, however, that in certain industrial applications, such utility cost data have been developed, typically in the form of loss of profit opportunities, and used as the basis for hopefully improved decision-making.

Prism

In conclusion, it is important to briefly mention PRISM (Program Reliability Information System for Management), which is an example of the new management controls covering the area of equipment performance and reliability. PRISM, as promulgated by the Navy in the Polaris program, involves the development of PERT—reliability networks, and associated quantitative data known as the RMI (Reliability Maturity Index), SCE (Schedule Compliance Evaluation), TQE (Technical Quality Evaluation), and RPM (Reliability Performance Measurement). The methodology by which these data are developed is too complex to cover in a paper of this length, but it should be emphasized that PRISM data will serve as another input to the PERT system and that the RMI phase of the program is now undergoing preliminary implementation at Raytheon's Missile and Space Division.

As a final note, it should be emphasized that the new PERT extensions, even though they will be showing up as a requirement in the not too distant future, must be approached with a degree of caution, and only after a thorough capability in PERT-time, i.e., network development and analysis, has been achieved throughout the organization. ■

ELECTROMAGNETIC PHENOMENA IN PLASMA DUCTS

by Paul A. Silberg

When the subject of weather phenomena is discussed by meteorologists or presented in meteorological texts, terminology such as high and low pressure areas and thermal mixing is often used. Only cursory mention, however, is made of the electrical phenomena accompanying severe weather. This paper points out these phenomena and suggests an explanation of them with particular reference to tornadoes.

Although there are many types of whirlwind activity that come, to one degree or another, under the general heading of tornadoes, the severe-storm tornado, *with the maximum speed of the whirling wind estimated to be as high as 500 miles per hour*, is one of the most violent storms that nature produces. On the axis of the whirling wind exists a partial vacuum which is responsible for lifting various heavy objects and carrying them, at times, for a considerable distance. The distinguishing characteristic of the tornado is a funnel-shaped cloud which is nearly always in evidence. When this cloud dips to the ground, the tornado is formed and its destructiveness is unleashed. This process may take place in steps and is generally accompanied by a loud roar. Occasionally, a peculiar whining sound is heard when the tornado cloud is high. Heavy hail, rain, and very intense lightning discharges frequently precede and accompany the tornado. The tornado moves across the ground at varying speeds ranging from a few miles per hour to between sixty and sixty-five miles per hour. It has been reported that the path length of tornadoes is shorter in the midwest than in the country as a whole. S. D. Flora[1] hazards a guess that this may be due to their lower moisture content there.

In addition, there are some features of a tornado that are seldom reported but occur in many, if not all, tornadoes:

1. Sferics occur before the tornado and increase in intensity during the tornado.
2. RF discharges on some occasions are formed close to and upon the ground and are often termed "peculiar lightning."
3. An odor described as "brimstone" is associated with almost all tornadoes.
4. Temperature of the surrounding air is reported to increase during the tornado.

[1]S. D. Flora, *Tornadoes of the United States* (University of Oklahoma Press, Oklahoma, 1954).

5. After the tornado, the temperature of the surrounding air is reported to become cooler.
6. The air near the funnel is described as hot.
7. Vegetation near the edge of the storm appears wilted—"looked burned or scalded."
8. The ground, trees, and tree stumps near the path of the tornado are dehydrated.
9. The axis of the funnel on occasion is reported to momentarily luminesce. This is sometimes called "fiery finger" or "tongue of fire."
10. The air close to the ground becomes momentarily still before the tornado begins.

Any complete theory of tornadoes must satisfactorily explain these features.

In a recent paper,[2] E. R. Rathbun discusses the initiation of a tornado by magnetohydrodynamic means but considers the actual tornado energy as aerodynamic (thermodynamic). The often-reported intense electrical activity is considered to be a second-order effect arising from the turbulent motion of the atmosphere. The role of the intense electrical phenomena (which have seldom been described in detail) is not well defined. Vonnegut[3] questions its association with tornadoes as: (1) fortuitous, (2) produced by the tornado, or (3) responsible for the tornado.

[2]E. R. Rathbun, J. Meteorol. *17*, 371-373 (1960).
[3]B. Vonnegut, J. Geophys. Research *65*, 203-212 (1960).

EYE WITNESS REPORTS OF TORNADOES

Most observers reporting on tornadoes confine themselves to describing the wind damage. Occasionally, careful observers also report on the electrical effects, including unusual luminous clouds, St. Elmo's fire, and fireballs. Several such reports are given here.

Alonzo A. Justice, "Seeing the Inside of a Tornado" published in *Monthly Weather Review*, 1930 (about a tornado at Greenbergh, Kansas, June 22, 1928).

"There was a screaming, hissing sound coming directly from the end of the funnel. I looked up, and to my astonishment I saw right into the heart of the tornado. There was a circular opening in the center of the funnel, about fifty to one hundred feet in diameter and extending straight upward for a distance of at least half a mile, as best I could judge under the circumstances. The walls of this opening were rotating clouds and the whole was brilliantly lighted with constant flashes of lightning, which zig-zagged from side to side."

S. D. Flora, *Tornadoes of the United States* published by University of Oklahoma Press, 1954 (view by Milton Tabor, editor of the Topeka Kansas Daily Capitol into tornado of March 23, 1913 at Omaha, Nebraska).

"We looked up into what appeared to be an enormous hollow cylinder bright inside with lightning flashes, but black as blackest night all round. The noise was like ten million bees plus a roar that beggars all descriptions."

Roy S. Hall, "Inside a Texas Tornado" published in *Weatherwise*, June 1951.

"As the tornado approached, the house grew dark but then was illuminated by an eerie blue light. Shortly after, the house was destroyed by the funnel, and Captain Hall found himself looking up into a tornado tube. The tube wall itself was "slick". Up inside the tube a luminous cloud floated in the axis. As the funnel passed by, 20-foot long streams of St. Elmo's fire were drawn from all the house wreckage to the funnel edge. Then the blue light disappeared."

Keith Bowker, published in *Osteopathic Magazine*, October 1953 (on the tornado at Flint, Michigan, June 8, 1953).

"A dancing funnel. An ugly black cloud with a ball of fire in it."

Stewartville, Missouri *News* about 1935.

"While carrying an umbrella, which obscured his view, a Mr. A. C. Hinderks was picked up by a tornado and dropped 75 yards away. He said that he was lifted above some telephone wires and that the atmosphere seemed as if it was on fire."

A. L. McCuiston, meteorologist in Tulsa, Oklahoma (unpublished observation of tornado cloud).

"I was on my way to my television weather show and was able to observe the tornado from about 5 miles distance. The main storm never reached the ground. However, straight winds were sufficient to cause considerable damage. I identified the storm as a "tornado" by the red glow in the cloud."

Major T. O. Haig, USAF "Tornado Aloft at Night," Oklahoma City, Oklahoma, 1959 (unpublished communication).

"After a tornado passed over Norman, Oklahoma and headed north, personnel at Tinker Field heard a sharp hissing sound overhead combined with a low-pitched continuous roar. We were conscious of an unusual and oppressive sensation. The noise source was definitely above us. When it was nearest us, I saw the sky above gradually grow lighter, then fade to black. The light was greenish in color. Associated with the light was a strong sensation of heat radiating downward. The noise increased in volume and then faded out as though it came from the south and passed us going north. The rain had stopped while this phenomenon was overhead."

Floyd C. Montgomery, "Some Observations on the Tornado at Blackwell, Oklahoma," published in *Weatherwise*, 1956.

" . . . As the storm was directly east of me, I could see fire up near the top of the funnel that looked like a child's Fourth of July pinwheel. There were rapidly rotating clouds passing in front of the top of the funnel. These clouds were illuminated only by the luminous band of light. The light would grow dim when these clouds were in front, and then it would grow bright again as I could see between the clouds. As near as I can explain, I would say that the light was the same color as an electric arc-welder but very much brighter. The light was so intense that I had to look away when there were no clouds in front of it. The light and the clouds seemed to be turning to the right like a beacon in a lighthouse . . . "

Lee Hunter's eye-witness report of the Blackwell, Oklahoma tornado in May, 1955 (as told to F. C. Montgomery).

" . . . The funnel from the cloud to the ground was lit up. It was a steady deep blue light—very bright. It had an orange-color fire in the center from the cloud to the ground. As it came along my field, it took a swath about 100 yards wide. As it swung from left to right, it looked like a giant neon tube in the air, or a flagman at a railroad crossing. As it swung along the ground level, the orange fire or electricity would gush out from the bottom of the funnel and the updraft would take it up in the air causing a terrific light—and it was gone! As it swung to the other side, the orange fire would flare up and do the same."

Mrs. Talmadge Veal, 1244 Macon Avenue, Macon, Georgia (private communication on the April 1954 tornado).

"The tornado struck at night, so I cannot testify as to the appearance (it unroofed my house). But it definitely was preceded by a curious kind of lightning—an incessant shimmer of light that had no thunder. I was in the bathtub and was unaware of this lightning because it was not accompanied by the usual thunder—until my daughter asked me to get out, as this electrical display (we thought it was lightning) had alarmed her. A few minutes later, the tornado struck."

1. Electrical Phenomena May Be Fortuitous

If the appearance of the reported electrical phenomena were fortuitous, then it would be expected that the tornado could occur with or without its presence. In general, some form of intense electrical activity is reported with *almost all* tornadoes. Even in the case of the dust devil, a dipole moment of the order of 10^9 esu has been measured.[4] Consequently, on a probabilistic basis, it would seem that more than coincidence of occurrence is involved. Moreover, the reported electrical activity is generally quite intense, and since the water or water vapor present has a high dielectric constant, some interaction with the intense nonuniform electrical fields present could easily take place. Such interaction would modify a purely turbulent flow pattern. The fortuitous occurrence of electrical phenomena is, therefore, highly unlikely.

2. Electrical Phenomena May Be Produced by the Tornado

It is probably correct to assume that the medium can be approximated as a very weak conducting fluid. Turbulent motion of a conducting fluid[5] may give rise to spontaneous magnetic fields which are quite strong. There are always small perturbations in any conducting fluid which can be accompanied by very weak electric and magnetic fields resulting from causes outside the fluid motion itself, e.g., magnetomechanical effect or thermal fluctuations. Are these perturbations amplified or damped by the turbulent motion in the course of time?

The time variation of the magnetic field perturbations, once they have arisen, is determined by two physical processes: (1) the dissipation of magnetic energy into heat by the induced currents, which tends to diminish the magnetic field, or (2) the magnetic effect of stretching the lines of force (i.e., when a fluid of sufficiently high conductivity is in motion, the lines of force move

[4]G. D. Freier, J. Geophys. Research *65*, 3504 (1960).

[5]Landau-Lifshitz, *Electrodynamics of Continuous Media* (Addison-Wesley Publishing Company, Reading, Mass., 1960).

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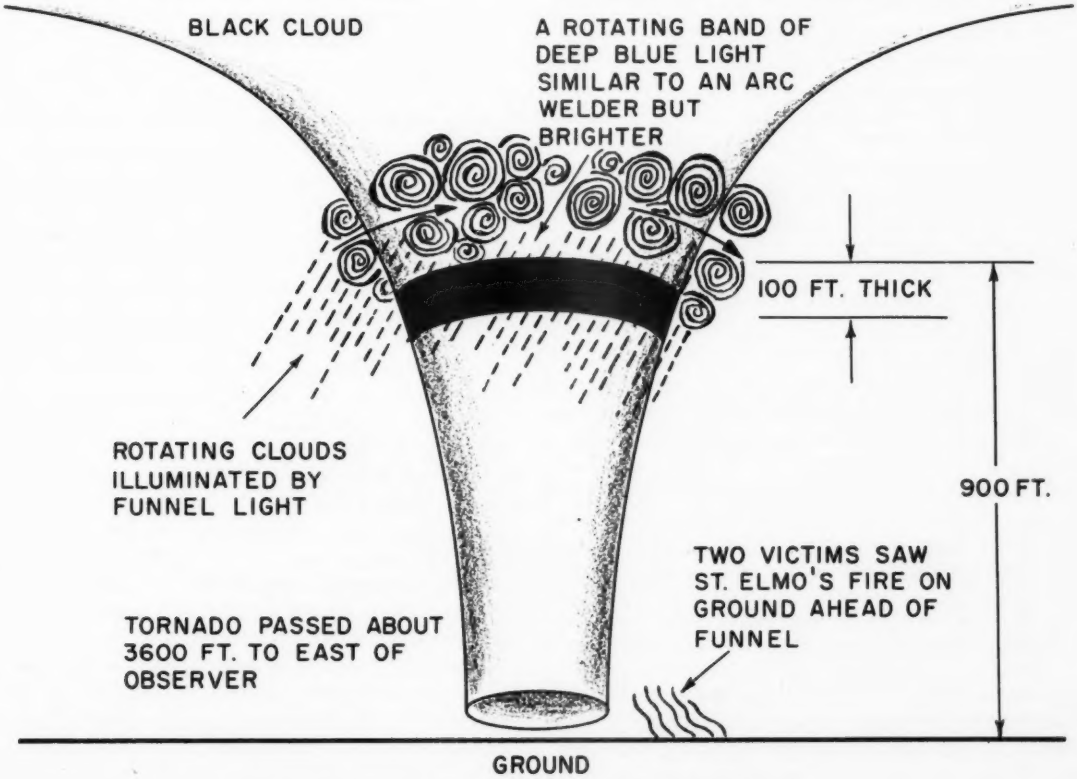

BLACK CLOUD
A ROTATING BAND OF DEEP BLUE LIGHT SIMILAR TO AN ARC WELDER BUT BRIGHTER
100 FT. THICK
ROTATING CLOUDS ILLUMINATED BY FUNNEL LIGHT
900 FT.
TWO VICTIMS SAW ST. ELMO'S FIRE ON GROUND AHEAD OF FUNNEL
TORNADO PASSED ABOUT 3600 FT. TO EAST OF OBSERVER
GROUND

Figure 1. View of tornado funnel at Blackwell, Oklahoma

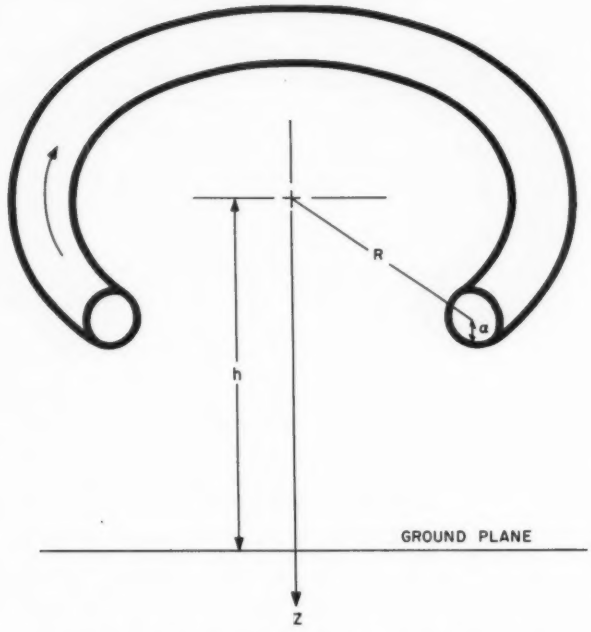


Figure 2. Ring current at altitude h above ground plane

as fluid lines and the magnetic field varies proportionately to the stretching at each point of each line of force) which tends to increase the magnetic field. In turbulent motion, on the average, any two neighboring particles move apart in the course of time with the result that lines of force are stretched and the magnetic field is strengthened.

If the conductivity σ of the fluid is less than $1/\nu\mu_0$ (where ν is the viscosity and μ_0 the magnetic permeability), losses in the fluid are too large to account for an increase of the magnetic field. However, if $\sigma > 1/\nu\mu_0$, then conditions are obtained for the spontaneous appearance of magnetic fields by the growth of small magnetic perturbations.

It can thus be seen that the magnetic field distribution must be similar to the turbulent distribution. Since the magnetic field moves with the fluid, the magnetic field vector rotates with the same angular velocity as the fluid. It is evident that the energy associated with this magnetic field must be less than the kinetic energy of the turbulent motion; consequently, any electromagnetic disturbance associated with the tornado is supported by the energy of the turbulent motion. Vonnegut[3] estimates that the power required to accelerate air continuously to a velocity of 250 meters per second over a circular area 100 meters in diameter is 10^{11} joules per second.

Raytheon's interest in weather phenomena dates back to early work done on weather radars. The investigation of severe weather phenomena is being pursued in the Advanced Development Laboratory at Wayland, Massachusetts. For example, a field study of ball lightning at Mt. Wittington, New Mexico was carried out in 1961 for the Air Force Cambridge Research Laboratories.

Since the maximum repetitivity of the tornado sferics is reported as 20 pulses per second,[6] it would be interesting to know, from measurements, the power dissipated per pulse and to estimate the energy associated with the turbulent motion within a tornado so that a more realistic evaluation of this concept is possible. Crude estimates appear to indicate that turbulence alone as a power source is not sufficient to generate the electromagnetic effects.

3. Tornadoes May Be Produced by Electrical Phenomena

Vonnegut has suggested an electrical theory for tornadoes which deserves further consideration.[3] He has estimated power requirements of the order of 10^{11} joules per second. Braham[7] estimated that an average energy of 10^{15} joules is stored in an average thunderstorm cell. From these numbers it is apparent that there is more than enough electrical energy stored in an average thunderstorm cell to sustain a tornado; moreover, from some reports of tornado activity it appears that the tornado cloud has a greater electrical activity associated with it than the average thundercloud. Electrical data associated with these storms are still too scarce for more reliable evaluation.

In order to seriously propose a tornado theory based on electrical activity within the tornado cloud, we must at least have some data that are indicative, if no more, that we are proceeding in a plausible direction. It is known first of all, that at least as much charge if not more is stored in the average tornado cloud as in the average thundercloud. Secondly, that if vortex motion occurs in a cloud having an appreciable amount of stored charge, currents are set up that are proportional to the velocity and the amount of charge. This conclusion is inevitable. The question that now arises is whether this current is large enough to have an appreciable effect on the fluid motions. It appears from the Blackwell tornado that huge currents are generated in at least some cases (see Figure 1). Clearly, in the upper portion of the funnel, there is a ring current of great physical extent and intensity. Moreover, the "tornado oscillator," reported by H. L. Jones at the Annual Meeting of the American Geophysical Union in 1960 as occurring in almost all tornadoes, appears to be a ring current.

Since it appears energetically feasible to generate a tornado by the electrical energy stored in the cloud, it may be tentatively assumed that the intense winds are caused by electrical energy which manifests itself as a ring current.

The sferics from this ring current have been reported to have the wave form of damped oscillations and are generated by a solenoidal electric field in a plane normal to its axis. This electric field also has the time

[6] H. L. Jones, Bull. Am. Meteorol. Soc. *32*, 380-385 (1951).
[7] R. R. Braham, Jr., J. Meteorol. Soc. *9*, 237 (1952).

response of damped oscillations and would, on the average, produce unidirectional motion of any charged particles present. For a sufficiently intense electric field, avalanche processes would enhance the residual ionization present and momentum transfer between the accelerated ionized particles and neutrals would produce a drift velocity in the field direction. Un-ionized particles such as water molecules (polar molecules can be accelerated by repetitious transient electric fields) would also contribute appreciably to the momentum transfer with neutrals.

In order to enhance any ionization present, neutral atoms or molecules must lose at least one electron thereby creating a positive ion and an electron. If the field is small, it would be expected that the free electron would attach itself to a neutral atom or molecule to form a negative ion. Then, the average sum of the drift velocities of both positive and negative ions would be zero unless some natural means were present to sweep one type of ion away from the other. For the present discussion we will assume that the fields are strong enough to reduce the probability of the occurrence of negative ions, and that the ratio of negative to positive ions is very small: $n_-/n_+ << 1$.

Since the time-varying magnetic field resulting from the ring current is most intense on the tornado axis, it is conceivable that gaseous breakdown might occur there for a sufficiently intense transient field strength. This could explain the fiery finger that is occasionally reported. Similarly, the hot air near the mouth of the tornado and the "scalded" vegetation that is often reported could result from the intense time-varying induced electric fields present. The "brimstone" color and odor could be traced to nitrogen oxides and ozone produced by the electric field.

Because the ring current is considered the essential mechanism of this tornado model, it presents a physically verifiable check point. This current, as reported by Jones, consists of repetitious damped oscillations with a repetition frequency of 10 to 20 cps and an oscillation frequency in the vicinity of 100 kc. The ring current in the tornado cloud could be formed as follows: If charge is stored in the cloud in an unsymmetrical manner, low-order thermal or pressure gradients could initiate the ring current by causing two or more of the charged regions to drift sufficiently close together so that breakdown between these regions would take place.

From the viewpoint of the conceptual theory, it appears that almost all of the properties of the tornado can be tentatively explained from the ring-current viewpoint. Let us consider the motion that could be imparted to neutral particles from their collision with charged particles being accelerated by a time-varying electric field with the time dependence of a damped oscillation. If there are charged particles in this electric field, they are accelerated and because of collisions with other particles their average velocity in the field direction soon reaches a stationary value. There are exact statistical methods that can be used to calculate current density and conductivity of a partially ionized gas in an electric field; however, for the present, we are only interested in the principles involved and simpler methods will serve equally well. Utilizing these assumptions, it is fairly straightforward to calculate the average particle velocity for an ionized gas and obtain a value of less than 1000 volts per centimeter to achieve velocities of the order of 500 miles per hour.

Consequently, it can be seen that a moderate electric field would be required to induce the maximum reported tornado velocities. It becomes worthwhile to consider the macroscopic vortical motion of a tornado and its relationship to the magnetic field produced by the ring current. For this, the Navier-Stokes equation for incompressible flow may be written as

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla P + \frac{\vec{f}}{\rho} + \eta\nabla^2\vec{v} \quad , \qquad (1)$$

where ρ is the mass density, P the pressure, η the coefficient of kinematic viscosity, and $\vec{f}$ the sum of external forces. Since we are assuming that induction forces contained in $\vec{f}$ dominate the motion, the viscosity term will be negligible except at the boundaries. At present we will ignore boundary effects. The external forces will then be electromagnetic in origin. The gravity force will be omitted but, because it can be represented as a gradient of a scalar potential, it can be incorporated into the first term representing the force generated by a pressure gradient. Thus, the general force term can be written as[5]

$$\vec{f} = -\frac{E^2}{2}\nabla\epsilon + \nabla\left[\rho\left(\frac{\partial\epsilon}{\partial\rho}\right)_T \frac{E^2}{2}\right] - \frac{H^2}{2}\nabla\mu \qquad (2)$$

$$+ \nabla\left[\rho\left(\frac{\partial\mu}{\partial\rho}\right)_T \frac{H^2}{2}\right] + (\epsilon\mu - \epsilon_0\mu_0)\frac{\partial}{\partial t}(\vec{E}\times\vec{H}) \quad .$$

Here ϵ and μ represent the electric and magnetic permeability. We assume that $\nabla\epsilon = \nabla\mu = 0$ and, approximately, that $\rho(\partial\epsilon/\partial\rho) = \epsilon - \epsilon_0$, as $\epsilon = \epsilon_0$ when $\rho = 0$. Since we will take $\mu \approx \mu_0$

$$\nabla\left[\rho\left(\frac{\partial\mu}{\partial\rho}\right)_T \frac{H^2}{2}\right] \approx 0 \quad , \qquad (3)$$

and we can write

$$\vec{f} = \frac{\epsilon - \epsilon_0}{2}\nabla E^2 + \mu_0\left(\vec{J}\times\vec{H}\right) \quad . \qquad (4)$$

It is easy to see that for highly polarizable matter (e.g., water vapor) the first term would contribute appreciably. However, its present inclusion would unnecessarily complicate the problem and so is omitted. In a more complete treatment, it must be included. Since

the flow is assumed incompressible,

$$\nabla \cdot \vec{v} = 0 \qquad (5)$$

and

$$\frac{d\vec{v}}{dt} = -\frac{\nabla P}{\rho} + \frac{\mu_0}{\rho}(\vec{J} \times \vec{H}) \quad . \qquad (6)$$

Along with the equation of motion we require Maxwell's equations

$$\nabla \times \vec{H} = \vec{J} \qquad \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\mu_0 \nabla \cdot \vec{H} = 0 \qquad \nabla \cdot \vec{D} = \rho^*$$

with

$$\vec{J} = \sigma\,[\vec{E} + \mu_0\,\vec{v} \times \vec{H}] = \rho^* e\vec{v} \quad . \qquad (7)$$

Since we have taken $\partial D/\partial t$ negligibly small, the charge conservation equation gives us

$$\nabla \cdot \vec{J} = 0 \quad . \qquad (8)$$

Upon substitution and combination, Maxwell's equations can be written in terms of the magnetic field vector as

$$\frac{\partial \vec{H}}{\partial t} - \nabla \times (\vec{v} \times \vec{H}) = -\frac{1}{\sigma \mu_0} \nabla \times (\nabla \times \vec{H})$$

$$= \frac{1}{\sigma \mu_0} \nabla^2 \vec{H} \quad . \qquad (9)$$

Also, if we take the curl of Equation 6, we obtain

$$\frac{\partial \vec{\Omega}}{\partial t} - \nabla \times (\vec{v} \times \vec{\Omega}) = \frac{\mu_0}{2\rho} \nabla \times (\vec{J} \times \vec{H}) \quad , \qquad (10)$$

with

$$\vec{\Omega} = \frac{1}{2} \nabla \times \vec{v} \quad .$$

Combining Equations 9 and 10 and assuming that $\nabla \sigma \sim 0$, $\nabla \rho^* \sim 0$, we obtain

$$\frac{\partial \vec{\Omega}}{\partial t} - \nabla \times [\vec{v} \times (\vec{\Omega} + \frac{1}{2} \mu_0 \frac{\rho^*}{\rho} e\vec{H})] = 0 \quad . \qquad (11)$$

If $\vec{\Omega}$ is almost stationary, $\partial\vec{\Omega}/\partial t \simeq 0$ and, to a good approximation, one solution is

$$\vec{\Omega} = -\mu_0 \frac{\rho^*}{2\rho} e\vec{H} \quad . \qquad (12)$$

This follows directly from the single-particle picture where,

$$m\Omega'^2 r = \mu_0 H' e\Omega' r \text{ or } \Omega' = \mu_0 \frac{e}{m} H' \quad . \qquad (13)$$

It must be remembered that the vorticity vector $\vec{\Omega}$ is defined as $1/2\ (\nabla \times \vec{v})$ and, for the case discussed ($v_z = v_r = 0$ and $\partial\, v_r/\partial_z$ small), is perpendicular to the velocity vector $\vec{v}$. Moreover, Equation 12 is not the only solution to Equation 11. Since our present purpose is to show in a simple manner that vortical motion is possible, no detailed attempt at examining the solutions to Equation 11 will be made.

For the purpose of further discussion, an analytic form for the ring current is considered. For simplicity, we will assume that the ring current lies in a plane parallel to the ground (see Figure 2). The damped oscillation frequency of the ring current is reported to be less than 1 mc, and the periodicity of the damping to be less than 100 cps. If the fields which comprise the ring current are slowly variable, a quasi-stationary approximation is permissible; otherwise electrodynamic analysis (retarded potentials) must be used. The criterion for this approximation is that the radiation path corresponding to the period of variation must be large compared to the dimensions of the oscillator [8]

$$c\,\tau >> l \quad ,$$

where c is the velocity of light (3×10^8 m/sec) and τ is the period. Since we are dealing with both damping periodicity and oscillation periodicity, we must consider the validity of the approximation for both. For damping we have

$$c\,\tau_d = 3 \times 10^8 \times 10^{-2} = 3 \times 10^6 \text{ meters} >> l$$

and for the oscillations

$$c\,\tau_0 = 3 \times 10^8 \times 10^{-6} = 3 \times 10^2 \text{ meters} \simeq l \quad .$$

Certainly the damping can be considered in the quasi-stationary approximation; however, this does not appear to be true for the oscillations.

Since we are primarily interested in the motion of a weakly conducting fluid in the time-varying magnetic field, and since $c\tau_d >> l$, we can use the quasi-stationary approximation for the magnetic field and obtain the space dependence.

Using this approximation,

$$\nabla \times \vec{B} = \nabla \times \nabla \times \vec{A} = \mu_0 \vec{J} \quad , \qquad (14)$$

where $\vec{A}$ is the vector potential, μ_0 the magnetic permeability, and $\vec{J}$ the current density. Since $\nabla \cdot \vec{B} = 0$,

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}}{r}\, d\tau \quad , \qquad (15)$$

where $d\tau$ is the volume element,

and

$$\vec{B} = \frac{\mu_0}{4\pi} \int i\, \frac{d\vec{l} \times \vec{r}}{|r|^3} \quad . \qquad (16)$$

We now assume that the current is uniform along $d\vec{l}$ and

[8] A. Sommerfield, *Electrodynamics* (Academic Press, Inc., New York, N.Y., 1952).

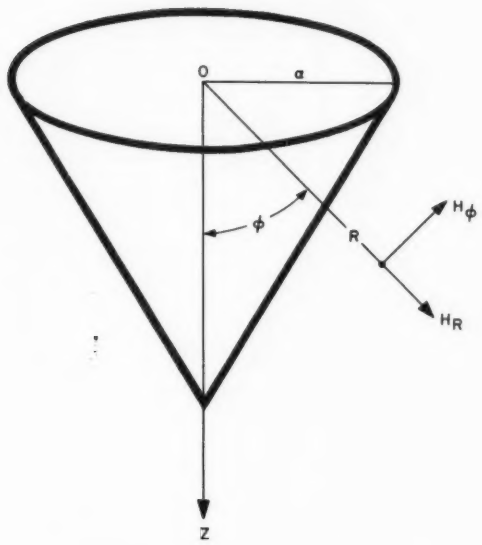


Figure 3. Magnetic field component of a circular current

obtain in spherical coordinates (R, ϕ) for $R^2 < a^2$ (see Figure 3)

$$B_R = \frac{\mu_0 i}{2a} \cos\phi \left\{ 1 - \frac{3}{4}\frac{R^2}{a^2}(5\cos^2\phi - 3) + \cdots \right\} \quad (17)$$

and

$$B_\phi = -\frac{\mu_0 i}{2a} \sin\phi \left\{ 1 - \frac{3}{4}\frac{R^2}{a^2}(5\cos^2\phi - 1) + \cdots \right\} . \quad (18)$$

These expressions can be written in terms of cylindrical coordinates (r, θ, z), where $R^2 = r^2 + z^2$ and, due to symmetry, $B_\theta = 0$. Thus

$$\begin{pmatrix} B_r \\ B_z \end{pmatrix} = \begin{pmatrix} \sin\phi & \cos\phi \\ \cos\phi & -\sin\phi \end{pmatrix} \begin{pmatrix} B_R \\ B_\phi \end{pmatrix} . \quad (19)$$

In terms of r and z, we obtain

$$B_r = \frac{\mu_0 i}{2a}\left[\frac{3}{2}\frac{zr}{a^2} + \cdots\right]$$

$$B_z = \frac{\mu_0 i}{2a}\left[1 - \frac{3}{4}(2z^2 - r^2) \cdots\right]$$

$$B_\theta = 0 .$$

If we put $B_0 = \mu_0 i/2a$ and retain the first two terms in B_z and the first term in B_r, we can write

$$B_r = B_0 \frac{3}{2}\frac{zr}{a^2}$$

$$B_z = B_0\left[1 - \frac{3}{4}(2z^2 - r^2)\right]$$

and

$$\vec{B} = [\vec{i}_r B_r + \vec{i}_z B_z]\, e^{-\beta t} \sin\omega t . \quad (20)$$

Since the supposed induced motion is perpendicular to the field lines, one would assume that the constant field lines derived from the field components would indicate the geometrical shape of the tornado. We can rewrite the field components for constant field lines as

$$1 - \frac{B_z}{B_0} = \frac{3}{4} a^2 (2z^2 - r^2) = \text{constant} .$$

This is a hyperbola with its vertex on the r, θ plane at $z = 0$. Its asymptotes are $z = \pm r/\sqrt{2}$. Clearly, this does not define the geometrical shape of the tornado. However, if we reproduce Equation 12

$$\vec{\Omega} = -\mu_0 \frac{\rho^*}{2\rho} e \vec{H}$$

it is easy to see that the shape of $\vec{\Omega}$ is also dependent on ρ^*, the charge distribution. Moreover, if it is supposed that ρ^* becomes smaller as the ground plane is approached from the cloud, larger values of $\vec{H}$ are necessary to maintain a constant value of $\vec{\Omega}$ and the vortical motion would tend to pull closer to the axis. However, with the extremely small amount of data available, further evaluation of these concepts would be fruitless.

Conclusion

Although the motion and electrical activity of a tornado are discussed from the viewpoint of magnetohydrodynamics, no definite conclusion can be drawn at the present time as to whether the electrical activity is responsible for the tornado or the tornado responsible for the electrical activity. It appears that the former could easily be true. Measurements in this area, however, are extremely scarce. It is hoped that the present paper suggests measurements to be taken in order to clarify these points so that a better understanding of tornado activity may be possible. ■

ENGINEERING SEMINARS CALENDAR

Seminars are held every other Tuesday at 5 P.M. in the cafeteria at the Executive Offices, Lexington.

OCTOBER 24
"New Programs and their Impact on the Technical Staff"
MARTIN SCHILLING—Vice President, Engineering and Research

NOVEMBER 7
"The Cybertron—A Self-Organizing Data Processor with Learning Ability"
RICHARD P. WITT—Manager, Advanced Development Department, CADPO

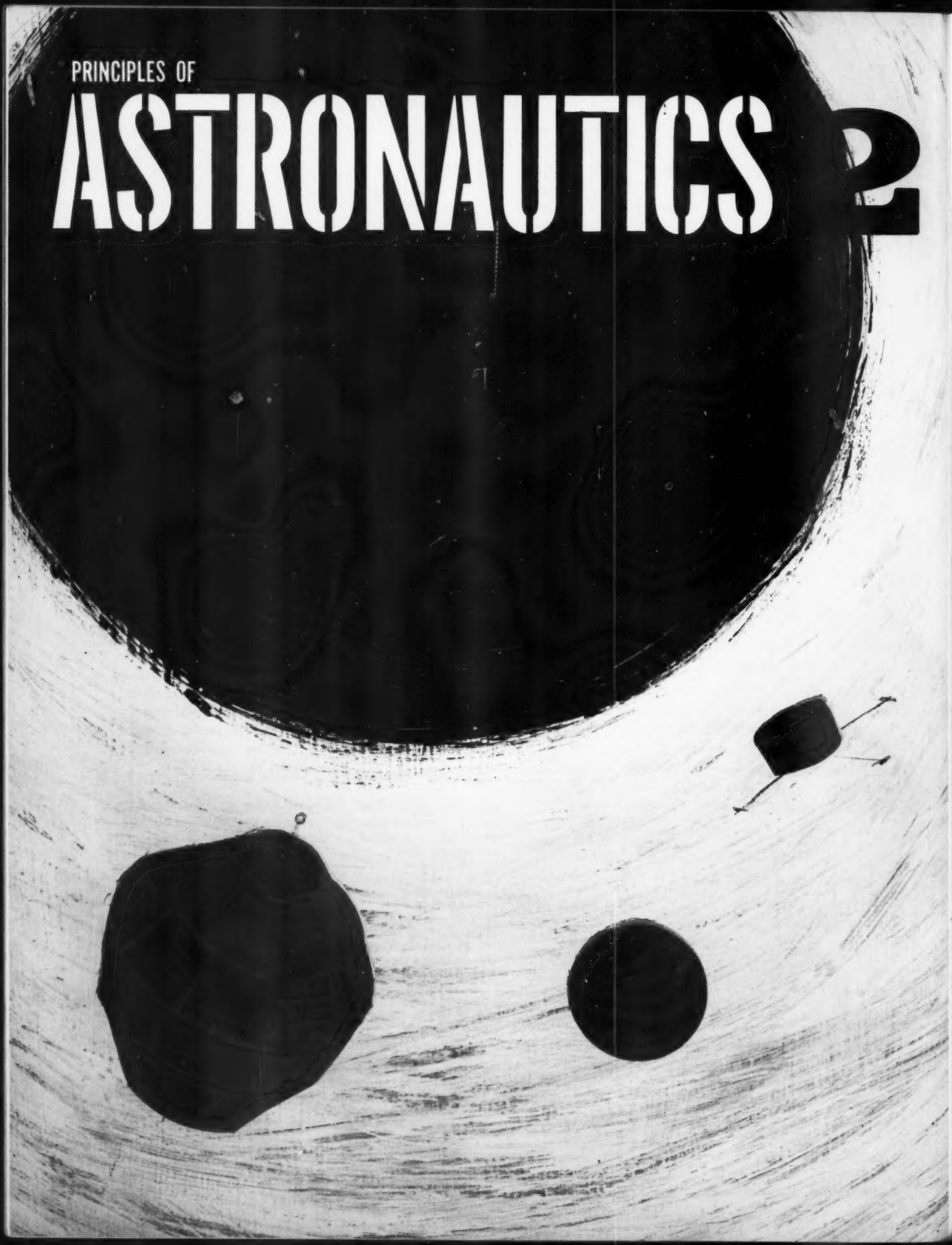
NOVEMBER 21
"Raytheon's Space Marketing Program"
JOHN LAZUR—Manager, Marketing Department, Missile and Space Division

DECEMBER 5
"Designing for the Interface between Men and Machines"
GUSTAVE J. RATH—Manager, Government Financial Analysis

DECEMBER 19
"Plasma Physics—Theory, Problems, and Applications"
E. ROLFE—Manager, Physics Department, SURANO

PRINCIPLES OF

ASTRONAUTICS 2



PLANETARY ARTIFICIAL SATELLITES

by Gerald A. Ouellette

ALL CELESTIAL BODIES with the exception of stars can be defined as satellites and all satellites obey the same physical laws which dictate their motion through space. In this sense, planets, comets, asteroids and some meteor groups are satellites of the Sun, whereas moons and debris such as Saturn's rings are satellites of the planets. The mathematical treatments of the motion of planets about the Sun or of moons about a planet are identical, except that in some cases, such as the Earth-Moon system, the perturbative terms are very large. In general, the Earth-Moon system is treated as a double planet system rotating about the barycenter (mass center), which traces out the solar orbit of this system. Generally speaking, the motion of small planetary artificial satellites is treated by classical techniques since relativistic effects can be neglected without introducing significant errors.

In discussing the motion of such satellites, the effects of other attracting bodies will be neglected and the mass of the satellite assumed to be too small to affect the parent planet. With these assumptions, the problem reduces to that of describing the motion of a particle mass about an attracting spheroid. For clarity of presentation, the basic motion about an ideal (spherical) earth will be shown, the perturbation terms due to the equatorial bulge and polar flattening (oblateness) will be superimposed. The effects of the Moon and Sun on a close satellite will be neglected although their long-term effects become appreciable for satellites whose mean orbital radius from the geocenter exceeds 20,000 miles. If satellites having very low density-to-volume ratios (Echo balloon) are analyzed, the effects of radiation pressure and pulsating atmospheres must also be considered. Since the analysis of these latter effects requires highly specialized analytical techniques, they will not be discussed further.

Basic Equations

The motion of an artificial satellite can, in general, be readily explained using Newton's laws of motion and gravitation, which he used to define the motion of the planets and the Moon. Further refinements to the form of his law of attraction are necessary to account for the oblateness of the Earth which results in non-central force terms.

Newton's law of gravitational attraction between two bodies is expressed as:

$$F = \frac{G(M_S + M_E)}{r^2} \quad , \tag{1}$$

where F is the force of relative attraction between the bodies. Since the mass of a satellite is much less than that of the Earth, it can be neglected in Equation 1. The acceleration produced by this force on the satellite is

$$a_S = \frac{G\,M_E}{r^2} \tag{2a}$$

and on the Earth is

$$a_E = \frac{G\,M_S}{r^2} \tag{2b}$$

It is apparent, therefore, that the acceleration of the Earth is negligible compared to that of the satellite and is assumed equal to zero. Equations 2a and 2b can be used to express the force exerted on each mass by the other as

$$F_E = F_S = \frac{G\,M_E\,M_S}{r^2} \quad , \tag{2c}$$

which is also a definition of Newton's law of force.

The equations of motion of the satellite in polar coordinate form are:
Radial acceleration

$$a_r = \ddot{r} - r\,\dot{\theta}^2 = -\mu^2 / r^2 \quad ; \tag{3a}$$

Tangential acceleration

$$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0 \quad , \tag{3b}$$

where the notation is shown in Figure 1 and $\mu^2 = GM_E$, the gravitational constant of the Earth. Equations 3a and 3b integrate to

$$r = \frac{p}{1 + \epsilon \cos\theta} \tag{4}$$

and

$$r^2\,\dot{\theta} = h = r\,v\cos\alpha \quad . \tag{5}$$

Equation 4 is that of an ellipse with prime focus at the

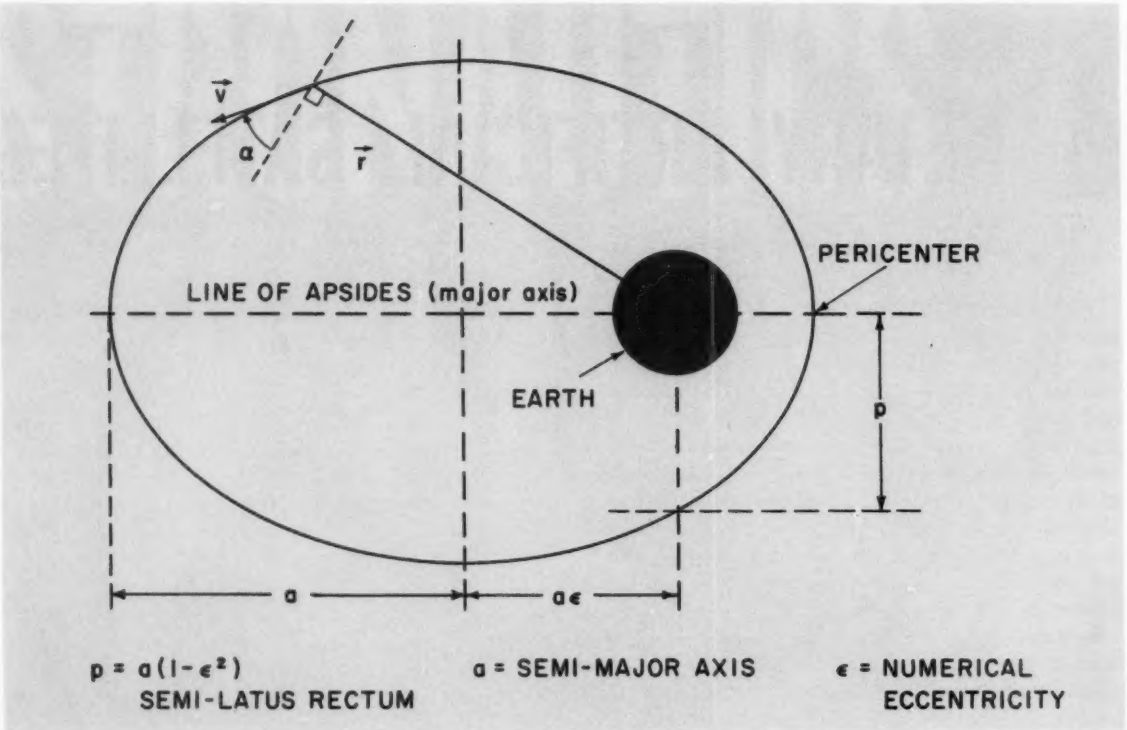


Figure 1. Elliptic orbit notation

geocenter. The quantities p, ϵ, and h are integration constants and are called the semi-latus rectum, the numerical eccentricity, and the angular momentum, respectively.

Motion under the influence of a central force is two-dimensional with the plane of motion defined by the initial velocity and radius vectors of the satellite. For values of ϵ equal to or greater than 1, the motion becomes parabolic or hyperbolic and the satellite escapes from the Earth as described in the first paper of this series (ELECTRONIC PROGRESS, July-August, 1961).

If Equations 3 through 5 are combined, the following vis-viva equation can be obtained:

$$v = \mu \left(\frac{2}{r} - \frac{1}{a} \right)^{1/2} \quad . \qquad (6)$$

This equation states that the orbital velocity of a satellite is a function only of the major axis of the orbit and the instantaneous magnitude of the radius vector. The major axis is defined in Figure 1. For circular orbits, $\epsilon = 0$, and the orbital radius r is constant and equal to the semi-major axis a. The orbital velocity in this case is constant and called circular or satellite velocity. Using Equation 6, this velocity is expressed by

$$v_s = \mu / \sqrt{r} \quad . \qquad (7)$$

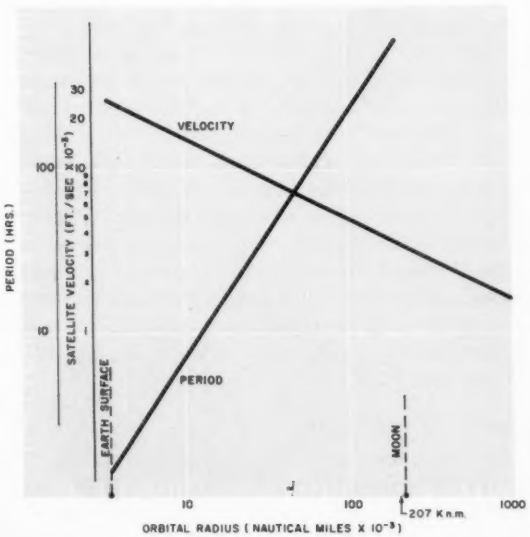


Figure 2. Satellite velocity and orbital period vs. orbital radius in cislunar space

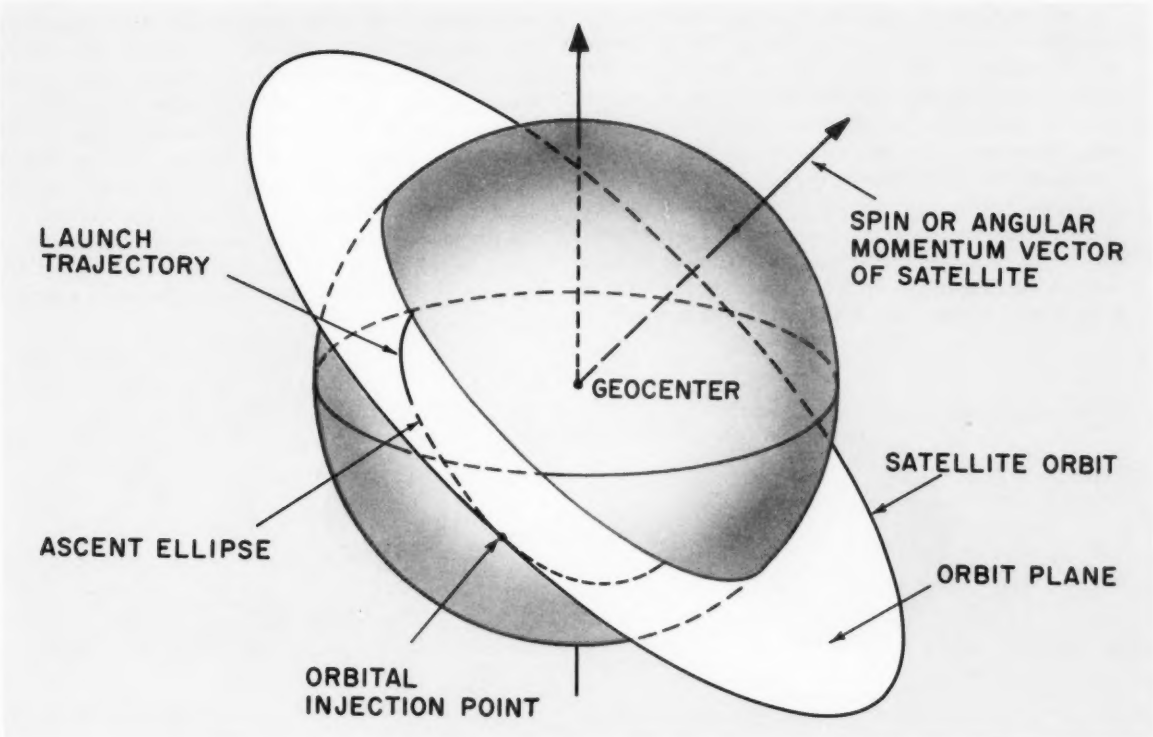


Figure 3. Typical satellite orbit

At the Earth's surface v_s is approximately 25,940 feet per second. Values of v_s versus orbital radius are shown in Figure 2. The length of time that it takes the satellite to make one complete orbital revolution is called the period and is equal to

$$P = \frac{2\pi a^{3/2}}{\mu} \quad . \tag{8}$$

For a circular orbit very close to the Earth, P is of the order of 80-90 minutes. Values of P versus orbital radius for circular orbits are shown in Figure 2.

Orbital Injection

To establish a satellite in a given orbit, certain initial conditions must be satisfied. Its subsequent motion is dictated solely by gravitational attractions when non-conservative perturbations are neglected. The initial conditions are completely specified by the radius and velocity vectors at the point where the satellite is injected into orbit. These conditions are that:

1. The radius vector lies in the specified orbit plane and its magnitude equals that of the orbital radius at injection; i.e., $\vec{r}_{injection} \equiv \vec{r}_{orbital}$, where the magnitude of $\vec{r}_{orbital}$ is given by Equation 4;
2. The velocity vector lies in the orbital plane, its magnitude being given by the vis-viva Equation 6, and its direction satisfying the angular momentum (Equation 5).

The technique normally used to inject a satellite into orbit is to launch it along a ballistic ascent trajectory whose apogee is tangent to the desired orbit. At apogee, thrust is initiated to accelerate the vehicle to orbital velocity after which subsequent motion is under free-fall conditions (see Figure 3).

Orbital Parameters

To describe an orbit in space, six parameters or elements must be specified. The most useful forms of these parameters are:

a = semi-major axis
ϵ = orbital eccentricity
Ω = right ascension of line of nodes
i = orbital inclination angle
ω = argument of perigee
θ = true anomaly or epoch at some given time.

These are illustrated in Figures 1 and 6. The first two parameters define the two-dimensional orbit, the next three the orientation of the orbit in space, and the last one, the position of the satellite in the orbit. For a circular orbit, ω is undefined since there is no perigee point, and true anomaly is redefined as the anomaly from some axis such as the line of nodes.

Orbital Perturbations

Earth satellites are subject to several types of perturbations, the more important being due to atmospheric resistance and oblateness of the Earth. The former removes energy from the satellite and causes the orbit to decay, whereas, the latter perturbs the shape and orientation of the orbit as functions of time.

Atmospheric drag removes energy from the satellite and transforms it into thermal energy. In the case of a circular orbit, the orbital radius decreases as more energy is given up, until the velocity falls below circular velocity and the satellite falls to Earth, as shown in Figure 4. It is interesting to note that as the satellite decays and its orbital radius is quasistatically reduced due to aerodynamic losses, orbital velocity increases in accord with Equation 7. This paradox—"atmospheric drag increases the orbital velocity"—is readily explainable because the energy change occurs primarily in the potential energy. It changes twice as fast as the kinetic energy, in the opposite sense, as a function of changes in orbital radius. This can be shown mathematically as follows:

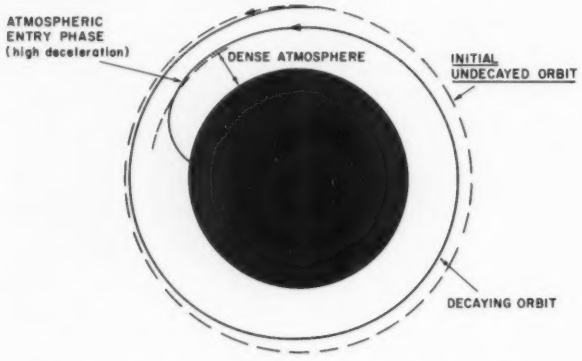


Figure 4. Typical decay from a low circular orbit

$$E = KE + PE$$

$$E = \frac{1}{2}v^2 - \frac{\mu^2}{r} = \frac{1}{2}\frac{\mu^2}{r} - \frac{\mu^2}{r} \quad , \qquad (9)$$

where E is the total energy per unit mass. For a quasistatically changing orbit

$$\frac{\partial\, KE}{\partial r} = -\frac{1}{2}\frac{\mu^2}{r^2} \quad ; \quad \frac{\partial PE}{\partial r} = +\frac{\mu^2}{r^2} \quad .$$

In the case of an elliptical orbit with perigee in the upper atmosphere, the satellite will suffer most of its energy loss near perigee. The altitude decay near perigee will be small for every orbital revolution since only a small amount of energy is removed; however, a small loss is strongly reflected in the major axis of the ellipse and apogee will change considerably. In general, highly elliptical orbits will become nearly circular prior to total decay as shown in Figure 5. The final orbit decay is similar to that illustrated in Figure 4.

The gravitational anomalies of the Earth result in non-central forces which perturb the orbit in several ways. By picturing the orbiting satellite as a one-particle gyroscope, it is apparent that the torque exerted by the mass contained in the equatorial bulge will produce a precession of the orbit plane (called regression of the nodes). In addition, the line of apsides (see Figures 1 or 3) rotates in the orbit plane, and the radius to the satellite varies harmonically at twice the angular frequency of the orbit. There are many other ways in which the orbit is perturbed, but these are small or are associated with high order terms in the Earth's potential field.

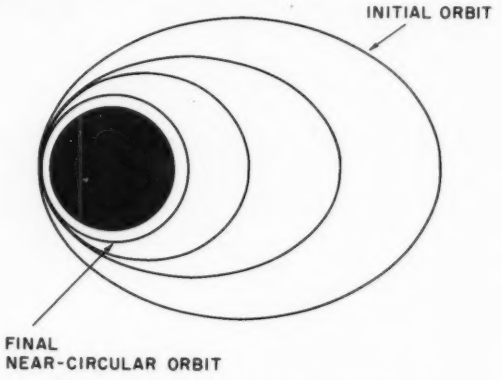


Figure 5. Typical decay of an elliptical orbit

For a truly spherical and concentrically homogeneous earth the gravitational potential would be expressed as

$$\Phi = \frac{g_e\, r_e^2}{r} \quad ; \qquad (10)$$

whereas for the Earth as we know it

$$\Phi = g\,R\left\{\frac{R}{r} + J\frac{R^3}{r^3}\left(\frac{1}{3} - \cos^2\theta\right)\right.$$

$$\left. + \frac{D}{35}\frac{R^5}{r^5}\left(35\cos^4\theta - 30\cos^2\theta + 3\right) + \ldots\right\} \qquad (11)$$

where g_e is the mean gravitational acceleration at the Earth's mean surface; r_e the mean radius of Earth; g the gravitational acceleration at the equator; R the equatorial radius; r the radius from geocenter to point in question; θ the co-latitude angle; J an Earth constant; and D an Earth constant. Using this latter equation and defining

$$\frac{1}{\bar{r}} = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{r}\,d\psi \quad , \qquad (12)$$

we have for low orbits (200-500 mile altitude) and nominal values of the earth constants:
Nodal Regression Rate

$$10.00\left(\frac{R}{\bar{r}}\right)^{7/2}\cos i \text{ degrees/day} \quad ; \qquad (13)$$

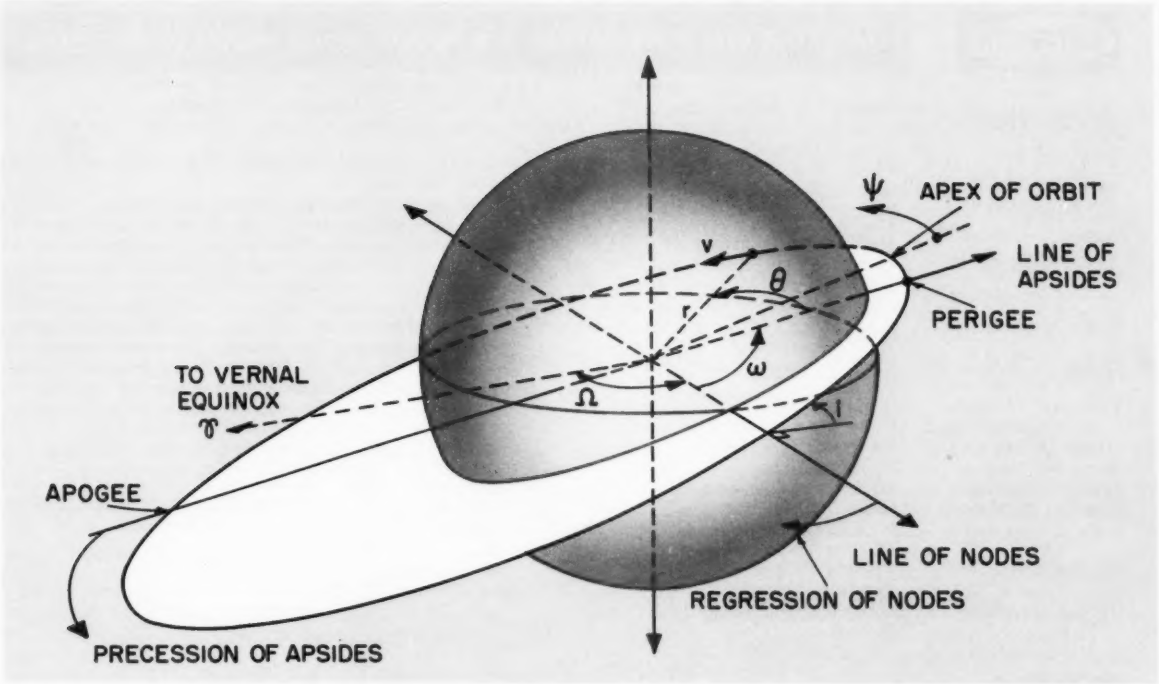


Figure 6. Geometry for motion about an oblate earth

Apsidal Precession Rate

$$5.00 \left(\frac{R}{\bar{r}}\right)^{7/2} (5 \cos^2 i - 1) \text{ degrees/day} \quad ; \quad (14)$$

Radial Perturbation

$$- 0.94 \frac{R}{\bar{r}} \sin^2 i \cos 2\psi \text{ nautical miles} \quad ; \quad (15)$$

Orbital Period (nodal crossing to nodal crossing)

$$P = 2\pi \sqrt{\frac{a^3}{gR^2}} \left\{ 1 - \frac{JR^2}{\bar{r}^2} \frac{7 \cos^2 i - 1}{4} + O(J^2) \right\}. \quad (16)$$

For typical orbit launch from the Atlantic Missile Range with an inclination angle i of approximately 35 degrees and a mean orbital radius of 300 nautical miles, the nodal regression rate (change in Ω) is approximately 6.1 degrees per day; the precession rate of the line of apsides (change in angle ω) is approximately 8.7 degrees per day; the maximum radial perturbation is 0.3 nautical miles; and the change in orbital period is 4.3 seconds as a result of earth oblateness. The effects of oblateness are minimized as follows: on orbital for polar orbits regression (i = 90°) as shown by Equation 13; on apsidal precession for orbits inclined 63.4 degrees as shown by Equation 14; on radial perturbation for equatorial orbits; (Equation 15) and on orbital period for orbits inclined 67.8 degrees as shown by Equation 16.

Summary

Under the influence of the Earth's gravitational field, a satellite traces out an ellipse with prime focus at the geocenter. In the absence of perturbations, this motion would continue indefinitely and the orientation of its angular momentum vector (Figure 3) would remain fixed in space. To establish the orbit it is necessary that the vehicle velocity and position vectors be identical with those of the orbit at thrust (rocket engine) termination.

In practice, the motion of a satellite deviates from that described above due to perturbing forces. A study of the perturbed motion leads to an understanding of the nature of the perturbing forces. Since near-Earth satellites are not strongly affected by the Sun, Moon and planets, it is possible to determine the shape of the Earth with considerable accuracy. In addition, as a satellite encounters atmospheric resistance, a knowledge of its aerodynamic behavior leads to information about the Earth's atmosphere at high altitudes.

The establishment of artificial satellites represents the first phase of an effort to study and exploit space and has as a by-product a better understanding of the world in which we live. ■

INVENTIONS

PATENT DISCLOSURES by the following personnel have been approved for filing:

GEORGE FREEDMAN	Bulk Diffusion Technique for Manufacture of High Frequency and High Power Transistors
ERNST SCHLOEMANN	Phonon Amplifier and Generator for Use at Microwave Frequencies Employing a Ferromagnetic Material
ERNST SCHLOEMANN	Adjustable Delay Line Using Spin-Wave Propagation
DUANE B. HAAGENSEN, PAUL A. SORENSEN	Electromagnetic Transparent Fluid Screens
EDWARD C. DENCH, JOHN M. OSEPCHUK	Beam Miser for M-Type Traveling-Wave Tubes
BARRY D. BROWN	Integrator Modulator
RICHARD E. LANG	Push-Pull Drive
JAMES C. DAVIS, JR.	Method of Linearizing "Raysistors"
WILLIAM M. HALL	Classified
JOHN M. OSEPCHUK	Multi-Stage Depressed Collector for M-Type Traveling-Wave Tubes
MICHAEL DANILOFF	Elastica Program Computer Without "Flop-Overs"
GEORG RUPPRECHT	Ferroelectric Signal—Translating Device

PATENTS

ABSTRACTS OF RAYTHEON PATENTS issued during July and August, 1961. The following are abstracts of the technical content of the patents and do not necessarily reflect the scope of coverage being claimed in the patents. Copies of the patents may be obtained through Raytheon's divisional libraries.

ROBERT A. FRYKLUND, "Automatic Remote Recorders," Patent No. 2,992,059

This patent relates to a radar recording system for transcribing PPI information from the face of a scope to a chart by the use of an optical scanning system which scans the face of the scope. The optical system converts the light intensity to electrical signals which are fed to a stylus and used to impress a permanent mark on the chart paper. The optical system is carried around the periphery of the PPI scope in synchronism with antenna movement and the recording chart is driven at a rate related to the motion of the vehicle carrying the system. In this way a permanent record of what has appeared on the PPI scope can be obtained.

RALPH R. URSCH, "Insulating and Cooling Devices," Patent No. 2,992,405

This patent relates to cooling and insulating electrical equipment such as, for example, transformers in which the apparatus generating the heat is mounted on a ceramic block to provide electrical insulation and a high thermal conductivity path to the heat sink.

JAMES H. SPENCER, "Method for Bonding Laminations," Patent No. 2,992,480

This patent relates to a method of aligning and bonding metallic laminae to form, for example, a magnetron anode. The laminae are stacked together with a bonding metal between adjacent laminae and are then tack welded along the edge of the laminae to form a rigid assembly. The tack welded group is then removed from the stacking jig and heated to braze the laminae together. This process eliminates having to keep the laminae stacked on the jig during the brazing operation, as well as eliminating the need for separate apparatus to compress the laminae during the brazing operation.

KENNETH E. BROWN, "Stabilized Pulse Generator Circuit," Patent No. 2,993,175

This patent relates to a wave-form generator which is stabilized with respect to supply voltage variations due to changes in pulse repetition rate, and overcomes any departure from linearity of the control (plate voltage) versus time (pulse length) characteristic of the phantastron-type circuit. This is accomplished by providing a circuit that appears as a linear load to the plate voltage supply and which also draws a constant current. The circuit uses a cathode follower in the screen-suppressor grid loop with a resistive load connected between the plate supply and the cathode of the cathode follower. The cathode of the cathode follower is coupled to the suppressor grid of the phantastron tube and the grid of the cathode follower is coupled to the screen grid of the phantastron tube.

CARL L. ANDERSON, "Communication Links," Patent No. 2,994,870

This patent relates to a packaging arrangement for use in communication links such as microwave relays for rural telephone service. A substantially weather-tight compact case is provided which houses the communication equipment and antenna in one enclosure and has independent vertical and horizontal adjustment features.

JOHN CATTERAL and JOHN A. PETERSON, "Display Cabinets," Patent No. 2,994,440

This patent relates to a display cabinet for merchandizing articles such as electron tubes. The cabinet utilizes a tiered display portion holding electrical articles to be sold mounted on a base which includes a simple electrical conductivity test circuit providing audible and visible signals indicative of the condition of the tube being tested.

FRANK E. TAYLOR and JOHN A. BUCKBEE, "Spiral Scanning Circuits," Patent No. 2,995,678

This patent relates to a spiral scan technique for use in reading out information on a storage device such as a storage tube. The spiral scans are interlaced by producing a recurring series of bursts of sinusoidal signals and a time-corresponding series of sawtooth signals, each sawtooth being delayed a different amount relative to its corresponding burst. The bursts and sawtooth signals are combined and applied in quadrature to the deflection coils of the storage tube. Use of interlaced spiral scans reduces the dead time formerly needed for the read-out operation.

JOSEPH H. APELBAUM, KENNETH W. DUDLEY, "Impregnated Cathodes," Patent No. 2,995,674

This patent relates to an impregnated cathode comprising a sintered body of tungsten containing chromic oxide and at least one alkaline earth oxide such as an oxide of barium, strontium, or calcium.

DAVID A. COPSON, BAYARD C. PIERCE, "Microwave Apparatus," Patent No. 2,997, 566

This patent relates to a microwave deep-fat fryer. The food to be cooked is placed in a wire mesh container which acts as an RF choke at microwave frequencies and the container and food are placed just below the surface of the cooking fat or oil which is heated by radiant heating means external to the microwave cavity. This construction provides deep-fat frying without causing the food to become soggy so that it maintains a desirable crisp appearance.

GEORGE H. KUNSTADT, "Off-Centered Sweep Amplifiers," Patent No. 2,998,542

This patent relates to an off-centered sweep amplifier useful with radar indicators in order to expand a specified

portion of the total sweep. A circuit responsive to a predetermined value of sweep voltage biases the sweep output amplifiers to cut-off during that portion of the off-centered sweep that is beyond the edge of the indicator tube. This action results in the electron spot returning to a center position in the indicator tube. Beam intensity is reduced during this period so that the spot is not visible.

PETER A. RIZZI, "Carrier Suppressed Ferrite Modulator," Patent No. 2,998,579

This patent relates to a ferrite side band generator in which the side band frequencies are propagated while at the same time suppression of the carrier frequency is achieved. An input signal is fed to a hybrid junction having two balanced arms which contain ferromagnetic material and which are terminated by signal reflecting means. A magnetic field is applied to each balanced arm to render the phase angle of signals propagated by the balanced arms substantially equal and to alternately increase the phase shift through one arm and decrease the phase shift through the other arm by substantially the same amount. The difference signal, which is a double side band carrier-suppressed signal, appears at the output arm of the hybrid junction.

JOSEPH L. LOVETT, "Audible Fish Alarms," Patent No. 2,998,591

This patent relates to a depth sounding apparatus which is provided with means to generate an audible signal when echoes are received from an object in the water. The audible signal is in addition to visible signals which appear on a scope. The audible signal is obtained by keying a multivibrator under control of the keying impulse of the depth sounder. The differentiated lagging edge of this multivibrator pulse triggers a gating pulse generator which produces a pulse having a duration corresponding to the range of depths between a point somewhat below the surface of the water and a point somewhat above the expected bottom. Any echo pulse occurring during the interval when the gate is open keys on an audio oscillator causing a loud speaker to emit a burst of sound.

PAPERS & PUBLICATIONS

All papers which have been presented at scientific meetings and published in technical journals are being listed in this section of ELECTRONIC PROGRESS. In order to ensure completeness, authors are requested to advise the Secretary, Technical Publications Panel (Lexington X240) when papers are accepted for presentation and publication.

Publications in Technical Journals

I. BRAUN, Research Division, "A New Method of Zone Leveling" published in *Solid-State Electronics*, September 1961.

W. C. BROWN, Microwave and Power Tube Division, "Power Transmission by Microwave Beam" published in *IRE Student Quarterly*, September, 1961.

R. ESPOSITO, J. A. MULLEN, Research Division, "Noise in Oscillators with General Tank Circuits" published in *1961 IRE International Convention Record, Part IV*, September 1961.

C. A. KLEIN, W. D. STRAUB, Research Division, "Carrier Densities and Mobilities in Pyrolytic Graphite" published in *Physical Review*, September 1, 1961.

C. A. KLEIN, P. P. DEBYE, Research Division, "On the Statistics of Multilevel Systems in Semiconductors" published in *Proceedings of the 1960 International Conference on Semiconductor Physics, Prague*, 1961.

W. RINDNER, J. M. LAVINE, Research Division, "Scribe-Plating and Etching" published in *Journal of the Electrochemical Society*, September 1961.

W. RINDNER, Research Division, "A Negative-Resistance Photoelectric Diode" published in *Solid-State Electronics*, September 1961.

F. STEUDEL, Equipment Division, "Counter Uses Complementary Transistors" published in *Electronic Equipment Engineering*, July 1961.

B. TANENBAUM, Equipment Division, "Dispersion Relations in a Stationary Plasma" published in *Physics of Fluids*, September 1961.

Papers Presented at Scientific Meetings

American Ceramic Society Basic Science Division Fall Meeting, Schenectady, N.Y., October 8-10, 1961

H. J. VAN HOOK, Research Division, "Equilibrium Studies in Ferrimagnetic Garnet Systems and Their Application to Single Crystal Growth."

Eastern Massachusetts Association of Professional Engineers and Land Surveyors, Lynnfield, Mass., October 4, 1961

G. A. OUELLETTE, Missile and Space Division, "Problems Associated with Space Maneuvers and Orientation in Space."

Electrochemical Society Semiconductor Symposium, Detroit, Mich., October 2, 1961

W. RINDNER, R. ELLIS JR., Research Division, "Electrolytic Etching of Germanium and Other Semiconductors in Water." J. J. OBERLY, Research Division, "Further Study by Electron Microscopy." W. RINDNER, J. M. LAVINE, Research Division, "Trace-Plating."

IRE Professional Group on Microwave Theory and Techniques, Cambridge, Mass., October 10, 1961

H. STATZ, Research Division, "Optical Masers."

IRE Merrimack Valley Subsection, North Andover, Mass., September 25, 1961

G. J. RATH, Executive Offices, "Teaching Machines and the Corporate Image."

IRE Chicago Section, Chicago, Ill., October 13, 1961

W. C. BROWN, Microwave and Power Tube Division, "Elements and Applications of Power Transmission by Microwave Beam."

International Atomic Energy Conference on Plasma Physics and Controlled Nuclear Fusion Research, Salzburg, Austria, September 4, 1961

W. A. JANOS, Equipment Division, "A Linearized Boltzmann Equation Analysis of the Runaway Effect."

International Conference on Magnetism and Crystallography, Kyoto, Japan, September 25-30, 1961

E. SCHLOEMANN, Research Division, "Theory and Application of Spin-Wave Instability in a Microwave Magnetic Field Applied Parallel to the DC Field." H. STATZ, M. WEBER, L. RIMAI, G. DEMARS, Research Division, G. KOSTER, M.I.T., "Exchange Interactions in the Paramagnetic Resonance Spectrum of a Concentrated Ruby."

Joint Industrial Electronics Symposium, Boston, Mass., September 20-21, 1961

R. P. CURTIS, Commercial Apparatus and Systems Division, "Trends in Ultrasonic Machining Techniques."

Twelfth International Astronautical Congress, Washington, D.C., October 2-7, 1961

B. A. KRIEGSMAN, M. H. REISS, Missile and Space Division, "Terminal Guidance and Control Techniques for a Soft Lunar Landing."

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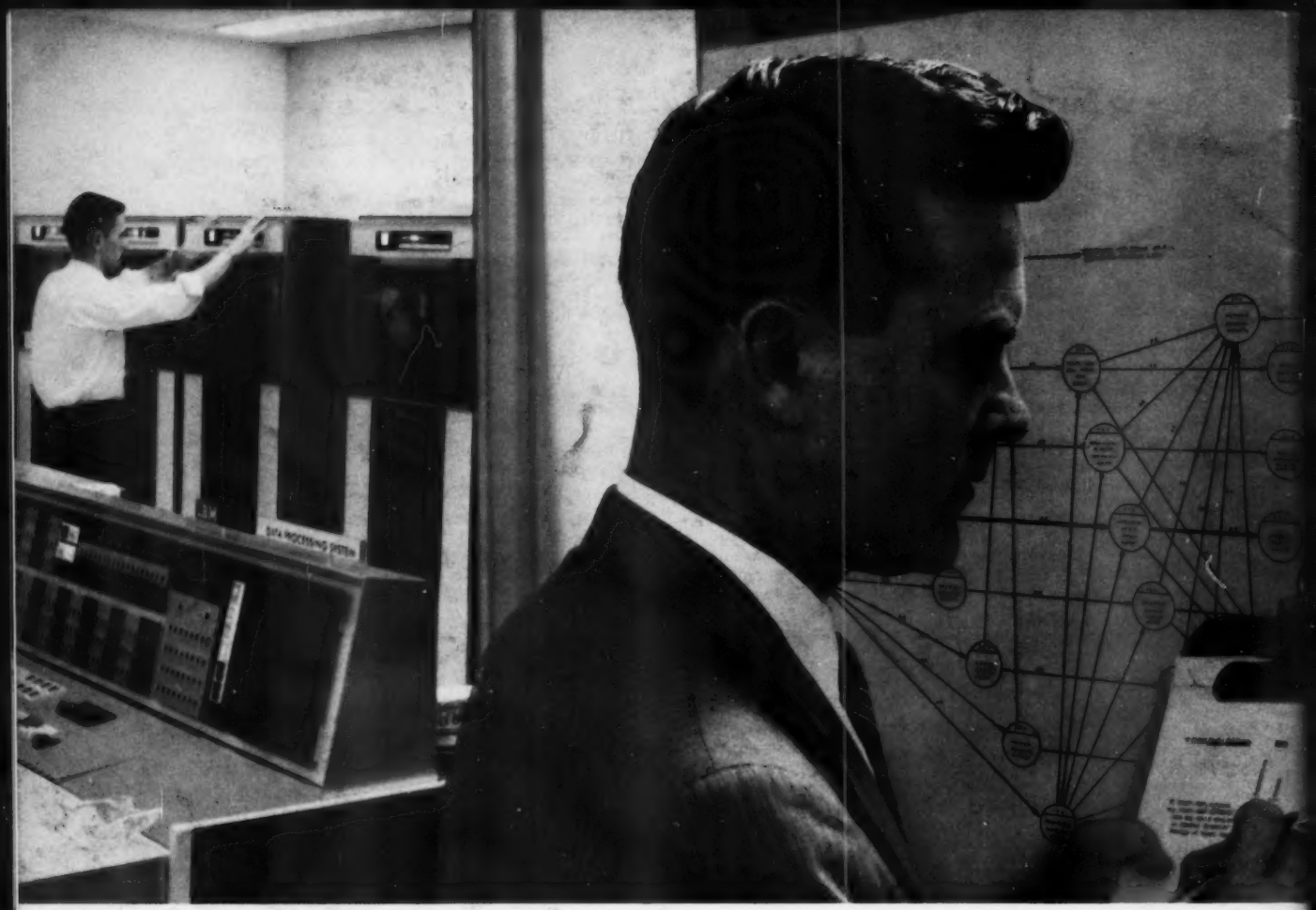

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